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Research Article

DETERMINATION OF MATHEMATICAL EQUATION MODEL ON THE 3D CURVATURE OF OCCLUSAL PLANES

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Abstract

In the present article, this writer has successfully developed a 3-Dimensional hyperbolic (parametric) equation for the oral occlusal surface of teeth in the dental arc. In fact, the patients' data do show that there are mainly two types of hyperbolic equation where one has a negative initial very large valued coefficient. The other has all very small positive valued coefficients. These two types of equation may be mapped to the General Relativity together with the Quantum Mechanics. To bridge the gap between them, this writer suggests that we may just employ the fuzzy time quantifier for the joining of these two major physics theories which are in fact can be considered as a U.S.A. commercial operational research dual pair etc. The implication of the present paper is the future development of an A.I. smart toothbrush.

Keywords: Quantum Mechanics, General Relativity, Oral Teeth Curvature, AI Smart Toothbrush, Fuzzy Logic.

INTRODUCTION

The occlusal surface of teeth in dental arch constitutes a plane called occlusal plane. This plane is important for function as well as for aesthetics. It usually follows a curvature that can be described in 3-dimensional (3D) including namely the curve of Spee and the curve of Wilson. These curves represent the sagittal and coronal planes respectively and they form a 3D sphere namely the curve of Monson. It is believed that the occlusal surface of teeth and collectively the occlusal plane, follow a 3D curvature and this 3D curvature can be presented by a mathematical equation model. By determining this mathematical equation model, the position of occlusal surface of any missing teeth can be predicted that will facilitate the clinical replacement of missing teeth.

Hypothesis:

Occlusal plane in a dental arch has a 3D curvature that can be represented by a mathematical equation model

Aim: To determine the mathematical equation model of the occlusal surface of dentition.

MATERIALS AND METHODS

Teeth model collection

Study participants with at least 12 pairs of occluding units include an intact maxillary right first molar and a stable maximal inter-cusp-al position (MIP) was assessed. The selection criteria were listed below (Table 1) [1]. Participants fulfilling the criteria were recruited, and written consent was obtained. This study had been approved by the Institutional Review Board of the University of Hong Kong and Hospital Authority Hong Kong West Cluster [3] (Reference Number: UW 20-848).

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Impressions were made with an irreversible hydrocolloid impression material (Aroma Fine Plus, Normal set; GC Corp) and stock metal trays (Coe Impression Tray; GC Europe) [4]. The impressions were then poured with Type III gypsum (Denstone KD; Formula SaintGobain) using the recommended powder to water ratio. The dental casts were digitized with a high accuracy (±5 mm) laboratory scanner (Trios D2000; 3Shape A/S). The digitized data were in standard tessellation language (STL) format and were converted into polygon (PYL) format using a 3D mesh processing software system (MeshLab v2021.07; Visual Computing Lab of the ISTICNR) [8].

Analysis of teeth model and fitting into a mathematical equation model "fitting procedure"

The occlusal surfaces of teeth models were captured using programming and numerical computing software (MATLAB R2022b; The MathWorks, Inc) (Figure 1).

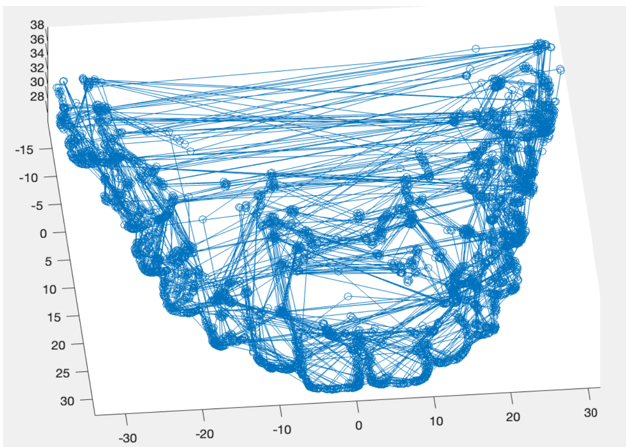

Figure 1. dB-Scan filtering of occlusal surfaces of teeth (model) image data

The numeric values of these occlusal surfaces were obtained with the following tableau spreadsheet data:

Inclusion Criteria	Participant Level	Participants with 12 or more functional occlusal units and stable maximal inter-cusp-al position (MIP)[2].
Exclusion Criteria	Tool Level Inclusion Criteria	Participants with maxillary right first molar tooth. Participants who score Grade 2 or less from Index of Treatment Need (IOTN) and have no ongoing or completed orthodontic treatment. Sound tooth or tooth with single surface restorations that still retain all natural occlusal landmarks. Participants with periodontal disease (Basic Periodontal Examination BPE Score 3) whereby there may be pathological tooth migration and alteration of occlusal plane. Participants under the age of 18 and unable to give consent. Teeth with extensive (2 or more surfaces) restorations that affect morphology.
	Tool Level	Teeth with pathological tooth movements such as fremitus, drifting, and over-erupt-ion.

Table 2. The first 15 sets of numeric values of these occlusal surfaces

Var1	Var2	Var3
-33.511784	39.605698	-7.553549
-33.426971	39.444866	-7.108828
-33.384308	39.261799	-6.800066
-33.56831	39.925034	-8.307693
-33.309647	39.10376	-6.519519
-33.725739	39.922184	-8.973909
-33.656193	39.742821	-8.202549
-33.762886	39.699162	-8.6419
-33.719566	39.541107	-8.249768
-33.885792	39.636368	-9.340449
-33.365986	38.466728	-7.295172
-33.230919	38.277882	-7.081221
-32.123001	36.96402	-5.324347
-33.315804	38.354614	-7.336343
-33.221924	38.222595	-7.384785

These data were then fitted to an appropriate mathematical equation model using MATLAB's library function ***Poly-fit-n***.

Validation of the fitting procedure

A curve was generated by the following equation using software MATHLAB 2022b.

Case Upper Cone:

Equation 1: $0.00143385279104214130x_1^2 + 0.202093451410087393*10^{-16}x_2x_1 + 0.13336762852347356\ x_1 - 0.12670874851400*10^{-15}x_2^2 + 0.0953389450830997576\ x_2 - 3.04130147171426657$

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Case Teeth Upper Cone:

Equation 2: $0.00143385279104214130x_1^2 + 0.202093451410087393*10^{-16}x_2x_1 + 0.13336762852347356\ x_1 - 0.12670874851400*10^{-15}\ x_2^2 + 0.0953389450830997576\ x_2 - 3.04130147171426657$

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The tip of multiple cones, which mimic the occlusal surfaces of teeth, were then fitted onto this curve with known mathematical equation model and a 3D model generated in PLY format (Figure 3). This multiple cone 3D file was then input into MATLAB, the numeric value of tip of cones were obtained and the mathematical equation was determined using

Poly-fit-n

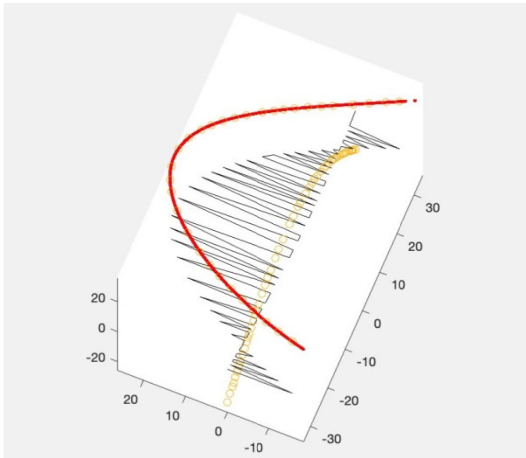

Figure 3. A fitting of the multiple 3D cones with the teeth model fitting equation

RESULTS

Analysis of teeth model and fitting into a mathematical equation model "fitting procedure"

A total of 168 teeth models were collected and analyzed in this study. The mathematical model generated were:

$$Z = [h_1 + r\,(\frac{e^{i\Theta}+e^{-i\Theta}}{2})]^2 + [k_1 + r\,(\frac{e^{i\Theta}-e^{-i\Theta}}{2i})]^2 + [h_2 + (\frac{e^{\alpha}+e^{-\alpha}}{2})] * [k_2 + (\frac{e^{\alpha}-e^{-\alpha}}{2})]$$

where Z is the height, h1, h2, k1, k2and r areconstants.

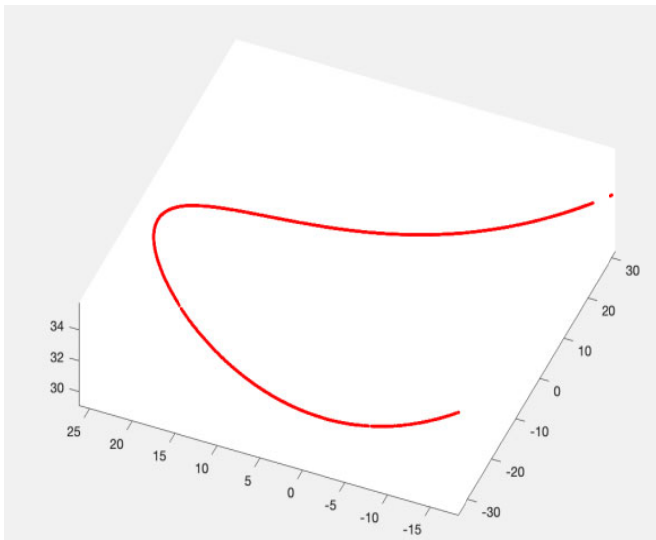

Figure 4. A practical visualization for the above teeth model fitting mathematical equation

Validation of the fitting procedure

The fitting procedure was able to generate the mathematical equation model that is used to align the multiple cones.

To get the wanted parametric mathematical equation model, I together with the two specialists have self-developed a dental arch dimension image-data processing procedure. It is like the following:

1. Import the original 3D teeth image file into the software MeshLab.
2. Filter, transform and rotate the image file by applying different functionality of the software to it;3. Save the actions as a script file (for the feasible automation of the large amount image files by python coding) in the corresponding computer folder.
3. Save the 3D teeth image as a pure text file without any normal data added to it.
4. Change the above 3D image file into spreadsheet file.
5. By calling the n-dimensional polynomial fitting function in the library of MATLAB, we may initially obtain the mathematical equation model like the following form:

$z = Ax_1^2 + Bx_1x_2 + Cx_2^2 + Dx_1 + Ex_2 + \text{constant}$ ------------- (7)

Precisely, if we want to improve the accuracy of the above equation (*) for the teeth vertex fitting, we may need to technically filter out the noise data of the imported spreadsheet image by calling the self-modified function that used the dB Scan function from the MATLAB library.

6. We have also developed a function that fits and calls the above self-modified dB Scan function for the imported teeth image spreadsheet data file.
7. Group and order the teeth image spreadsheet data file.
8. Save the fitted curve of the image teeth into either the jpeg or figure file.
9. We may then remodel the above fitted image file by using CAD software with cones fitted to the previously obtained curve and save it as a suitable format.
10. Convert the above suitable format of the into pure text data file by software MeshLab.
11. Change the pure text cone fitted file into a spreadsheet format.
12. Repeat steps 7 – 9 once more.
13. Check the calculated oral arch dimension curve with the one computed in step 6 and determine whether the coefficients of the two fitted equations are completely the same.

DISCUSSION, APPLICATION & CONCLUSION

How we may convert the original computer-generated equations into the wanted general mathematical model equation:

For the general oral arch dimension equation, we previously get:

$z = A\,x_1^2 + B\,x_1\,x_2 + C\,x_2^2 + D\,x_1 + E\,x_2 + \text{constant}$

According to the commutative and associative property of mathematical addition method in Abstract Algebra, the equation may be reformed into [6], [7]:

$Z = A\,x_1^2 + C\,x_2^2 + D\,x_1 + E\,x_2 + \text{constant} + B\,x_1\,x_2$ --------- Commutative Property

$Z = A\,x_1^2 + C\,x_2^2 + D\,x_1 + E\,x_2 + \text{constant} + B\,x_1\,x_2$

$= (A\,x_1^2 + C\,x_2^2 + D\,x_1 + E\,x_2) + \text{constant} + B\,x_1\,x_2$
$= A\,x_1^2 + C\,x_2^2 + D\,x_1 + E\,x_2 + (B\,x_1\,x_2 + \text{constant})$
--------- Associative Property

But

$A\,x_1^2 + C\,x_2^2 + D\,x_1 + E\,x_2 + con_1$ is an equation of circle while$B\,x_1\,x_2 + con_2$ is an equation of a hyperbola,
and

hence, we may express the equation Z in parametric form:
$Z = (h_1 + r\cos\Theta)^2 + (k_1 + r\sin\Theta)^2 + (h_2 + \cosh\alpha)(k_2 + \sinh\alpha)$

Or in its exponential form:

$Z = [h_1 + r(\frac{e^{i\theta}+e^{-i\theta}}{2})]^2 + [k_1 + r(\frac{e^{i\theta}-e^{-i\theta}}{2})]^2 + [h_2 + (\frac{e^{\alpha}+e^{-\alpha}}{2})]*[(k_2 + (\frac{e^{\alpha}-e^{-\alpha}}{2})]$

Geometrically, by the Pythagoras Theorem, we may further assume that:

$[h_1 + r(\frac{e^{i\theta}+e^{-i\theta}}{2})]^2 + [k_1 + r(\frac{e^{i\theta}-e^{-i\theta}}{2})]^2 = c^2$
i.e. $Z = c^2 + [h_2 + (\frac{e^{\alpha}+e^{-\alpha}}{2})] * [(k_2 + (\frac{e^{\alpha}-e^{-\alpha}}{2})]$
$= c^2 + ab$ where $a = [h_2 + (\frac{e^{\alpha}+e^{-\alpha}}{2})]$ and $b = [(k_2 + (\frac{e^{\alpha}-e^{-\alpha}}{2})]$

Or in the format of a conjugate expression (Pell's Equation):

$Z = (c + \sqrt{ab} + \sqrt{2c\sqrt{ab}})*(c + \sqrt{ab} - \sqrt{2c\sqrt{ab}})$

(With a combination of different addition and subtraction up to a maximum four sets of equation expression; also, with the negative sign in front of the equation,

i.e. $[-(c + \sqrt{ab} + \sqrt{2c\sqrt{ab}})*(c + \sqrt{ab} - \sqrt{2c\sqrt{ab}})]$

or a maximum of eight sets of Pell's equation expression)and may be considered as a full version of the 3-dimensional hyperbola curve which has been shown only a half part in figure 4.

$Z = [c + \sqrt[4]{ab}\,(\sqrt[4]{ab} + \sqrt{2c})]\,[c + \sqrt[4]{ab}\,(\sqrt[4]{ab} - \sqrt{2c})]$ (with a combination of different addition and subtraction up to a four sets of equation expression & optional simplification & may be a depth and wide previous study in the number theory & solve Pell's Equation which is NOT the core focus of the present paper; also with the negative sign in front of the equation,

i.e. $[-(c + \sqrt{ab} + \sqrt{2c\sqrt{ab}})*(c + \sqrt{ab} - \sqrt{2c\sqrt{ab}})]$

or a maximum of totally eight sets of Pell's equation expression) [4]

Moreover, we may consider the equation $[(c + \sqrt{ab} + \sqrt{2c\sqrt{ab}})*(c + \sqrt{ab} - \sqrt{2c\sqrt{ab}})]$ as a special case of the hypergeometric series which we may construct the corresponding hypergeometric differential equation with the solutions at the singular point. This may give us a total of $\binom{6}{3}$ linear relations or connection formulas. There are also Kummer's 24 solutions with Q-form where the Schwarz triangle maps give us a Monodromic group with integral formulas. Then we may have Gauss's contiguous relations plus the Gauss's continued fraction for the transformation formulas at those points like Z = 1, -1, 1/2 & some other points etc [5].

From the elementary computer programming results, there are two types of Pell's equation:
I: A positive signed leading coefficient of the equation with extremely small values for all the other coefficients and

II: A negative leading sign coefficient of the equation with extremely large values for all the other coefficients.

If we carefully interpret the equation sets I as the theory of quantum mechanics and the equation sets II as the theory of Einstein's Theory of General Relativity, then we may find that they are just the mirror image of each other and hence can't be unified. Or they both existed in a mirror with either one part as the real object and the other part as the imaginary image of the object. That is my elementary theory for the fundamental relationship between our quantum mechanics and general relativity which can be analogy by a mirror. That is in the commercial operation research sense, quantum mechanics and general relativity are just the dual pair of each other. Thus, mathematically, there is NO way to unify the ordinary vector space and its dual vector space. But the ordinary vector space is isometric to its double dual vector space etc. In fact, as the major difference between the quantum mechanics and the general relativity is staying only in the method of time assignment: absolute Vs relative, this writer suggests that we may employ a fuzzy time quantifier as a bridge among these times to overcome such conflicts. This is just like the bounded state and the unbounded state in the quantum solid chip such that we may also introduce some fuzzy methods to handle or solve the above quantum well problem.

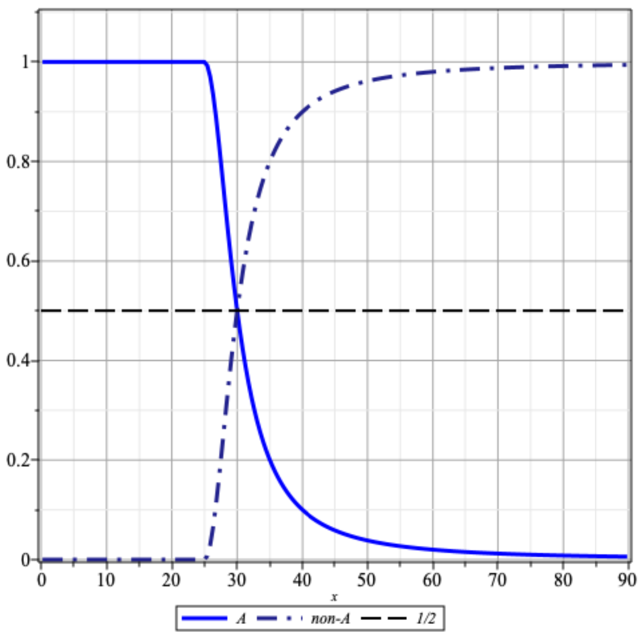

Fig.4.4 Affiliation functions

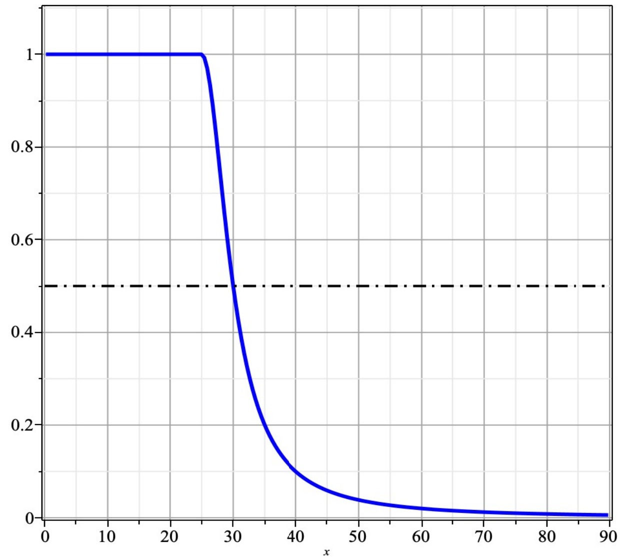
Fig 4.3 Affiliation function of the fuzzy subset A="Fuzzy Time (Boundary Value Problem)"

The below Affiliation functions tell us how the dual pair of the quantum mechanics and the general relativity problem may be solved by the fuzzy time quantifier for the "Absolute" and "Relative" times interaction. (N.B. These figures are generated by the Canada Maple Soft Personal Version 2024.) In practice, my proposed fuzzy time quantifier equation may be:

mu_A := piecewise(25 < x, 1/(1 + ((x - 25)/5)^2), 1)

Or

$$mu\_A := \begin{cases} \dfrac{1}{1+\left(\dfrac{x}{5}-5\right)^2} & 25 < x \\ 1 & otherwise \end{cases}$$

which is just the core one for the generation of the above two figures: 4.3 & 4.4. This writer wants to note that a wide and depth research will be needed for the further study in the topic of bridging the quantum mechanics and general relativity. The aforementioned equation may be only the start of such research.

In brief, there are still lots of rooms for the study of linking or the conversion (i.e. the proposed fuzzy time quantifier) between the quantum mechanics and the general relativity or the vice versa, this writer just wants to start or "heat the fire head" for the such kind of area in the present suggested research.

2. What will be the application of the present teeth curvature 3D plane surface MATLAB programming project?

The core of the present teeth curvature research project is to process those teeth data by computer programming. My key idea is to input the associated data into the program with the MATLAB library pre-installed file named "DB-Scan" and hence, applying the "If-then-else" loop to technically categorize and select these inputted data according to the distance or length between the outsiders and the major centered data with various output. To go a further step, we may then classify the defects of the studied teeth data with reference to the situations or disease just like the tartar or the case in the figure 5 below. Therefore, we may go ahead for the development of the AI smart toothbrush which may monitor and provide advice to their everyday users.

To conclude, what may we get from the research project in brief?

In brief, from the present teeth curvature research project, we may finally get the most wanted generalized mathematics model equation in the simplified form like the following:

$Z = c^2 + ab$

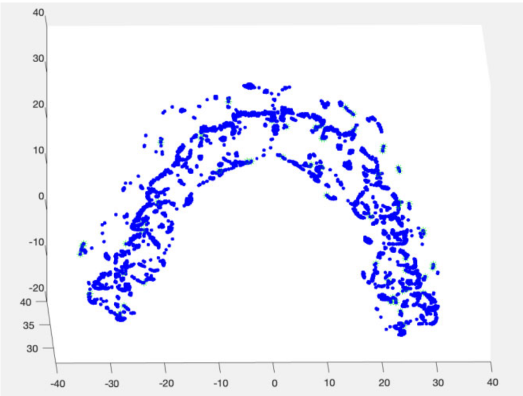

Figure 5. In practice, categorization of teeth data according to the distance between the major centered distribution and the outsiders

where $a = [h_2 + (\frac{e^{\alpha}+e^{-\alpha}}{2})]$, $b = [(k_2 + (\frac{e^{\alpha}-e^{-\alpha}}{2})]$
and $c^2 = [h_1 + r\,(\frac{e^{i\theta}+e^{-i\theta}}{2})]^2 + [k_1 + r\,(\frac{e^{i\theta}-e^{-i\theta}}{2})]^2$

Or in the conjugate expression:

$Z = (c + \sqrt{ab} + \sqrt{2c\sqrt{ab}})*(c + \sqrt{ab} - \sqrt{2c\sqrt{ab}})$
(with a combination of different addition and subtraction up to a four sets of equation expression; also with the negative sign in front of the equation, i.e. $[-(c + \sqrt{ab} + \sqrt{2c\sqrt{ab}})*(c + \sqrt{ab} - \sqrt{2c\sqrt{ab}})]$ or a maximum of totally eight sets of Pell's equation expression) and may be considered as a full version of the 3-dimensional hyperbola curve which has been shown only a half part in the previous mentioned figure. Moreover, we may further technically categorize those teeth defects such as tartar attached on the surface of the everyday users' teeth when they are brushing and hence, develop an AI smart toothbrush to monitor and provide advice about users' teeth for such AI smart toothbrush users (N.B. We always assume the addition for the above general equation as subtraction is only a special case by adding the negative signs.)

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