

**A REVIEW TO UNDERSTAND WHAT SUPERCONDUCTIVITY IS THROUGH ITS NEW REDEFINITION
AND A PATH TO ESTABLISH ITS COMPLETE THEORY*****José Félix Estanislau da Silva**

Master in Applied Theoretical Physics from the Institute of Physics of São Carlos (IFSC) at the University of São Paulo (USP), Brazil

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Abstract

Currently, the theory of superconductivity has been the subject of conversation and controversy in prestigious universities, large laboratories and companies involved in technology and scientific innovation. This interest is largely motivated by the possibility of constructing an experimental setup that exhibits its transition temperature corresponding to that of the Earth's ambient temperature and pressure. In this way, this new technology could replace many of the electronic components in use in civilization, reduce their cost-benefit ratio and become more efficient, as well as having many other scientific uses. For these reasons, in this article, this scientific researcher intends to show the new trends in the theory and experiment of superconductivity at the beginning of the 21st century, through an original summary about its developments and experimental advances. Furthermore, originally and in conclusion, propose a simple theoretical path for constructing a phenomenological theory, which could explain almost all or all experimental results and predict others. In this sense, a broad and new definition of superconductivity may be necessary, but one that does not reject its definition initiated with the experimental studies of Heike Kamerlingh Onnes, *et al.*

Keywords: Superconductivity, Definition, Experiment, Theory, Temperature, Pressure, Ambient, Earth.

1. INTRODUCTION

From the beginning of the third decade of the 21st century to the present day, some researchers or scientific groups have published articles claiming to have achieved a material arrangement that exhibits superconductivity at room temperature and pressure or at Earth's ambient temperature. In this direction, for example, a group of scientists from the experimental field of superconductivity argued in the magazine *Nature* [[1], 2020], that they had set up the carbonaceous sulfur hydride arrangement, where superconductivity could arise at Earth's ambient temperature, but under great pressure. However, for these same scientific publications, a note of retraction was demanded by another large part of the scientific community of the theory of superconductivity and other scientific areas [[2], 2022]. As a result, superconductivity has become the subject of more careful discussions and controversies in the scientific circles of prestigious universities, scientific laboratories and large technology and innovation companies. This interest in developing a material arrangement that exhibits a superconducting transition temperature corresponding to that of the Earth's temperature and pressure has various motivating causes. In this sense, this new technology could replace almost all the electronic components in use in a civilization's electrical grid system, reducing its cost-effectiveness and increasing its efficiency. In terms of transportation, a train could be built, whose speed could be much greater than that of ordinary trains. This is possible through the levitation originated by the superconductor. Electric current bobbins, power generators, end so on, without energy losses and more efficient, could also be manufactured using superconductor technology. To know more about these practical applications and others, the observant reader can consult the reference [[3], 2024], for example, where there are various figures, images and projects,

which stimulate interest in this problem that is being studied and developed continuously. In relation to the academic environment, a new voltage unit (V_{90}) has already been introduced, which replaces the old voltage unit (V). This new unit of measurement was originated from the scientific work published by Brian David Josephson in 1962 in the area of superconductivity [[4], 1962]. In this article, in theory, he proposed the real possibility of making a junction consisting of two or more superconductors (Josephson junction), defending that Cooper pairs can exhibit the behavior of the Tunnel effect. Josephson junctions have great potential for practical applications because they are related to frequencies of the order well above 1 GHz. In this sense, for example, they can be used to build equipment capable of detecting magnetic fields of intensity of 10^{-15} *Tesla*, which are the so-called magnetic fields of great insensitivity of measurement. By this means, the magnetic fields generated by planet Earth and the human brain, which are respectively of the order of 10^{-6} *Tesla* and 10^{-13} *Tesla*, could be measured with great precision. To read about the intensities of the magnetic fields of the human brain and planet Earth and their relationships, the observant reader can consult references [[5], 2018], [[6], 2019] and [[7], 2021].

What has been asserted in the preceding paragraphs is the motivation for writing this article. Besides giving the observant reader an extended review, which condense and explains the area of superconductivity in a single article. Additionally, in the conclusion, this scientific researcher proposes a simple scientific path to elaborate the possible Theory of superconductivity, which would explain almost all or all of the experimental results discovered up to now, in addition to predicting other behaviors of a superconductor. In this sense, a broad and new definition of superconductivity is perhaps necessary, but does not reject its definition initiated with the experimental studies of Heike Kamerlingh Onnes, *et al.*

*Corresponding Author: *José Félix Estanislau da Silva*,
Master in Applied Theoretical Physics from the Institute of Physics of São Carlos (IFSC) at the University of São Paulo (USP), Brazil.

As this article is essentially a review, the method used to produce proofs and arguments is a vast collection of articles, bibliographies, and scientific experiments already published in this area. In what follows, this article is divided into 4 parts, which are subdivided and, at the end, an original and stimulating conclusion is exposed. Now, the observant reader by himself can become conscious of the content of each part of this article.

2. Part 1

But then, what is superconductivity? How a superconductor was invented by mankind? In this Part 1, through the highlighted questions themselves, a study is made concerning the origin of superconductivity, its first definition or concept, its implications and applications.

2.1 What happens to the resistance of a metal when its temperature approaches absolute zero?

The question in this subsection was one of the issues raised by scientific researchers from the mid-19th century to the beginning of the 20th century. During this period, branches of Classical Physics Theory, for example, Thermodynamics, Electromagnetism, and so on, were used to explain the behavior of the interior of matter. However, new concepts were needed to explain the behavior of the interior of matter observed through experimental arrangements, which were beyond the theory at that moment. Thus this is the question discussed in this subsection 2.1.

2.1.1 Then, what did William Thomson think and research about this question?

At this time, the properties of heat were one of the favorite studies of William Thomson (1824-1907), who was named Lord Kelvin by the scientific community. He studied the discoveries of Jacques Alexandre César Charles (1746-1826) with respect to the variation of the volume of gases as a function of temperature variation. Substantiated on experiments and simple calculations, Charles had concluded that all gases would have zero volume when the temperature reached $-273.15\text{ }^{\circ}\text{C}$ [[8], 1787]. However, Lord Kelvin proposed that the volume of matter did not become zero at this temperature, but rather the kinetic energy of its molecules [[9], 1882].

In his scientific research, renewing science, Lord Kelvin defined absolute zero temperature as the lowest temperature that bodies can reach. This temperature corresponds to a point where molecular agitation is zero. In other words, at absolute zero temperature (0 K), the molecules of bodies are completely at rest. Thus, 0 K corresponds to $-273.15\text{ }^{\circ}\text{C}$ on the Celsius scale. As for the concept of heat, Lord Kelvin defined that it corresponds to energy in transit from one body to another due to the difference in temperature. In Classical Mechanics, this transfer always occurs from the body with the higher temperature to the body with the lower temperature, until they reach thermal equilibrium. By this definition, heat can be called thermal energy in motion. Then, about the **resistance** and conductivity of a metal as its temperature approached absolute zero, Lord Kelvin proposed that the electrons themselves must be at rest. In other words, the electrons should be frozen at absolute zero temperature. Consequently, the electronic conductivity in a metal should tend to zero as its

temperature approaches absolute zero. This being the case, he strongly suggested that the **resistance** of metal should be of infinite value at absolute zero temperature, otherwise conductivity would arise [[9], 1882].

In a nutshell, conceptually, the resistance of a metal was related to the scattering amplitude of electrons by the atomic and molecular ions of the lattice inside the metal. Therefore, until 1907, the scientific proposal for the behavior of the resistance of a pure metal was as follows: it should decrease with decreasing temperature, as indicated by experimental results, present a minimum value in the neighborhood of absolute zero temperature and assume an infinite value at this zero, as proposed by Lord Kelvin. In this sense, Fig. 01, at the top, graphically displays this behavior for the observant reader, besides two other behaviors that differ from each other.

However, at the end of the 19th century and beginning of the 20th century, theoretical advances, for example, the wave-particle duality concept and new experimental results of measurements of physical properties to describe the interior of matter, for example, specific heat¹, led to the emergence of the Theory of Modern Physics². Subsequently, this theory was compiled to lead to the development of the Theory of Quantum Mechanics³ to explain the behavior of the interior of matter. For this reason, some scientific researchers had been disagreeing with the proposition defended by William Thomson (Lord Kelvin). Some of them defended that the resistance of a metal should be constant when its temperature reaches absolute zero. Others defended that the resistance of a pure metal should be zero at that temperature.

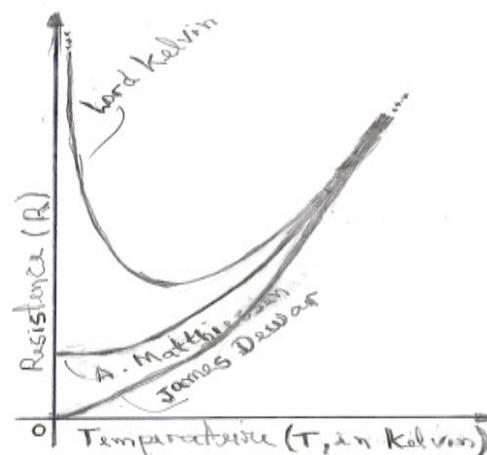


Fig. 1. Graphical solution to the problem of the resistance (R) of a metal as a function of its temperature approaching absolute zero (0 K). At the end of the 19th century and beginning of the 20th century, there was the solution defended by Augustus Matthiessen (1831-1870), [[20], 1864]. This solution was the graph between the other two graphs in the figure. The graph at the top of the R versus T coordinate was the solution given by Lord Kelvin (1824-1907), [[9], 1882]. The other graph, closer to the T coordinate, was the solution proposed by James Dewar (1842-1923), [[21], 1897].

¹ About the specific heat, the observant reader can consult references [[10], 1780], [[11], 1819], [[12], 1906] and [[13], 1912] among many others.

² About the foundation of Modern Physics, for example, references [[14], 1901], [[15], 1958] and [[16], 1961] or many others equivalent to these can be consulted.

³ Concerning the foundation of Quantum Mechanics, for example, references [[17], 1928], [[18], 1935] and [[19], 1949] can be consulted among many Others.

Then, in a graphical summary, the problem of the resistance of a pure metal, at absolute zero temperature, had three possible solutions, as follows in the graph given in Fig. 01.

Thus, this problem crossed the 19th century into the 20th century, and Heike Kammerlingh Onnes became aware of its scientific existence. Then, what follows in the next subsection is a discussion of the actions of this scientific researcher with respect to the resistance of a metal in the neighborhood of absolute zero (0 K).

2.1.2 What did Heike Kammerlingh Onnes do to analyze the resistance of a metal at absolute zero temperature?

Heike Kammerlingh Onnes (1853-1926) and his assistants at the University of Leiden, in their laboratory, made their first experiments measuring resistance versus temperature in thin platinum and gold wires [[22], 1907]. In these experiments, they used a Hydrogen liquefaction process to produce a temperature close to absolute zero. In this way, from 1906 to 1908, they were able to obtain experimental measurements of the resistance for these metals and others under a minimum limit temperature of **14 K**. These experiments showed that the resistance behaved decadently and almost linearly in relation to the temperature, which decreased until reaching this minimum limit. At this limit, the resistance assumed an almost constant value. These results contributed to inducing Heike Kammerlingh Onnes in the direction of the proposal theoretically defended by Lord Kelvin for the behavior of the resistance of a metal as a function of the decrease in its temperature to absolute zero, as shown at the top of Fig. 01.

Additionally, however, scientific researchers knew that the resistance of metal wires depended on the quantity of chemical and physical impurities present within these metals. In this sense, Kammerlingh Onnes and his assistants carried out some experimental arrangements to measure the resistance of gold as a function of the addition of silver-type impurities [[22], 1907]. In these experiments, they were able to observe that the intensity of the resistance was proportionally dependent on the increase in the concentration of silver and independent of the temperature in the vicinity of absolute zero. Thus, these results strongly suggested that the resistance of a metal became of little intensity, that is, tended towards zero, for a pure metal subjected to a temperature close to absolute zero. Therefore, producing the purest metal possible was convenient for carrying out experiments to measure its resistance as a function of temperature.

At that moment, the interest was to reduce the liquefaction temperature, **14 K**, of the experiment's cooling substance. In this direction, on July 10, 1908, in the laboratory of the University of Leiden, Kammerlingh Onnes and his assistants managed to liquefy Helium [[23], 1909], whose minimum temperature, **1.1 K**, was closest to the vicinity of absolute zero. Simultaneously, in Leiden, quantum phenomena were observed through experimental measurements of the specific heat of some solids as a function of decreasing temperature. Through these measurements, a decrease in specific heat was observed for temperatures tending towards absolute zero. This fact contributed to Kammerlingh Onnes modifying⁴ some

containers of his experiment with the goal of accelerating the process of measuring the resistance of platinum as a function of temperature, which was initially at a gauge temperature of 14 K.

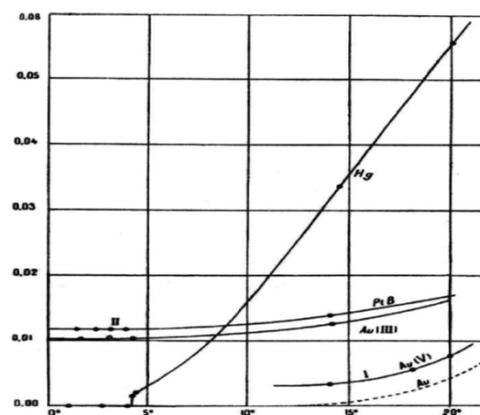


Fig.2. Graphs of the resistance of some metals versus temperature in degrees Kelvin [[24], 1913]. Here, the resistance of platinum (Pt) and gold (Au) are being compared with the resistance of mercury (Hg). The index I and II, respectively, mean that the metal's cooling temperature was that of liquid Hydrogen and liquid Helium. Other indexes mean the presence of impurities inside the metal

In this direction, experimental arrangements to measure the resistance of platinum versus temperature were carried out by Kammerlingh Onnes and his assistants [[25], 1910]. In these experiments, Helium was liquefied to obtain a temperature closest to the vicinity of absolute zero. On December 2, 1910, experimental measurements of the resistance of platinum as a function of decreasing temperature indicated that the resistance of this metal wire remained constant for temperatures lower than **4.5 K**, as exhibited in Fig. 02. This constant residual value of resistance was attributed to impurities contained in the platinum sample. By this means, Kammerlingh Onnes proposed that the resistance of platinum, without the presence of impurities in its interior, would decrease to a value of zero at the minimum temperature of liquid helium, **1.1 K**, and, consequently, at absolute zero temperature.

Prior to this result, Kammerlingh Onnes and his assistants had carried out experimental arrangements to measure the resistance of mercury as a function of decreasing temperature. In these experiments, the resistance of mercury, at a temperature of **14 K**, still showed a large inclination towards zero, that is, it did not remain constant. In addition to this fact, in the Leiden laboratory they had a lot of practice in purifying mercury by distillation. Then, in April 1911, Kammerlingh Onnes and his assistants carried out experimental arrangements with the objective of measuring the resistance of pure mercury versus the decrease in temperature, but using the liquefaction of helium to reach temperatures close to absolute zero [[26], 1911]. All the results obtained from these experiments, taken together, showed that the resistance of pure mercury abruptly went to zero at temperatures lower than **4.3 K**, as exhibited in Fig. 02. In addition, the resistance of pure gold with these experimental arrangements was measured and its value tended to zero, not abruptly, for temperatures lower than approximately **4.2 K**, as exhibited in the same figure.

⁴ The experimental setup with the transfer of liquid helium failed because the extra heat capacity of the platinum resistor caused drastic boiling and rapid evaporation of the liquid helium that had been transferred. Heike Kammerlingh Onnes decided not to change the liquid helium transfer system.

However, he decided to expand the original liquefier to accommodate the platinum resistor, which was a bit large compared to the other components. Thus, this experimental arrangement was very insensitive to measurement (typical of experiments in Quantum Mechanics)

On June 20, 1912, Kammerlingh Onnes suggested to one of his collaborators the possibility of making an alloy consisting of mercury (Hg) and gold (Au) [[27], 1913]. In this direction, they carried out some experiments to measure the resistance of this alloy as a function of the decrease in temperature with the liquefaction of Helium. In December 1912, the results obtained showed that the resistance of this alloy went to zero in the same way as had occurred with pure mercury, however, for a transition temperature greater than **4.2 K**. In a complementary study, the resistance of other metals⁵ as a function of the temperature of Helium liquefaction was measured by the group from Leiden University. The results obtained exhibited the same behavior, but this occurred from different transition temperatures.

Thus, Kammerlingh Onnes finished his main experimental measurements on the behavior of a metal's resistance when its temperature tended to absolute zero (0 K).

2.1.3 Then, what conclusions can be argued after the experimental measurements made by Heike Kammerlingh Onnes in respect of the resistance of a metal in the proximity of absolute zero temperature?

In subsection 2.1.1, the author of this article made a preliminary analytical summary of the origin of the question of the behavior of the metal's resistance in the vicinity of absolute zero temperature (0 K). In that subsection, the graphs in Fig. 01 show three behaviors for the resistance at that temperature. These predictions were proposed differently by scientific researchers, because other Physical Theories were being developed at the end of the 19th century. Of these predictions, the proposal supported by Lord Kelvin represented explanations for the behavior of matter's interior through the Theory of Classical Physics and advances in Thermodynamic Theory. Onward, in subsection 2.1.2, a scientific analysis was focused on the experimental measurements carried out by Kammerlingh Onnes and his assistants. From these measurements of the resistance of a metal as a function of temperature reaching the limit of absolute zero, the author of this article can affirm that:

(1) Heike Kammerlingh Onnes began his experimental arrangements by taking the position of an observer present in a classical observation referential⁶. But later, before obtaining conclusive results, he changed his observation referential. He became present in a non-classical⁷ referential to set up his experimental arrangements and observe their results. This attitude occurred because of advances in the Theory of Modern Physics, which progressed to become the Theory of Quantum Mechanics among others. Quantum Physics was developed to carry out experimental arrangements for measuring discrete and almost insensible quantities of the behavior of the interior of matter, at that moment, and other experimental and theoretical possibilities;

(2) After many experimental arrangements to measure the resistance of pure and non-pure metals as a function of decreasing temperature towards the vicinity of absolute zero, Heike Kammerlingh Onnes was led to conclude that the resistance of pure metals, from a critical transition temperature, tended towards the value zero. Consequently, Lord Kelvin's proposal for the behavior of resistance tending to infinity, in the limit of 0 K, was gradually rejected and his argument based on Classical Physics was also rejected;

(3) A new scientific phenomenon was developed, which was given the name **supra** conductivity. Later, the name was changed to **super** conductivity⁸;

(4) The interest in resistance tending to zero intensity, from a critical temperature, T_C , had two faces: a purely scientific face and a purely technological one, because there was great potential for the manufacture of technological mechanisms for common use. For example: bobbins made of superconducting material capable of producing intense magnetic fields (1.10^7 Gauss, which corresponds to 10 Tesla), persistent currents, trains moving by levitation, public electricity grid systems founded on superconductivity [[3], 2024], and so on; and finally;

(5) Because of the difficulties in the experimental arrangements, to find out whether the resistance of pure mercury had, in fact, gone to zero abruptly from the critical temperature of 4.2 K, Heike Kammerlingh Onnes and his assistants carried out experiments increasing the temperature until reaching this critical value, where the resistance suddenly reappeared⁹. This procedure suggests a **reversible** process of the procedure they were doing: decreasing the temperature to the vicinity of 0 K and measuring the resistance of the metal. However, in Thermodynamics, the occurrence of some processes or events are **irreversible**. Then, in principle, a question could be formulated as follows: Was this particular experiment an **irreversible** or a **reversible** event?

3. Part 2

At the beginning of the 20th century, with the breakthrough of superconductivity, a new phenomenon was measured using experimental arrangements, but there was no explanation for these results in the content of condensed matter. Therefore, a new theory had to be developed to explain this behavior of the interior of matter. In this sense, this is the objective expressed in this Part 2 of this article and its subsections. Then, the beginning of Part 2 is as appears at the top of subsection 3.1.

3.1 Is there perpetual current in a superconductor?

After having carried out the experiments that showed superconductivity, Heike Kammerlingh Onnes became interested in knowing the intensity of an electric current in a superconducting state (close to the critical temperature, T_C). Then, he and his assistants built an experimental setup consisting of two bobbins. One of the bobbins was consisted of spirals of lead wire, was placed inside a double-walled Dewar flask and immersed in liquid helium at $T = 1.8$ K. The other

⁵ For example: the resistance of tin and lead tended to zero, respectively, for temperatures lower than the transition temperatures of **6 K** and **4 K**.

⁶ The observer who is only conscious of the Theory of Classical Physics is present in a classical observation referential. In this way, his experimental arrangements will indicate results that are resonant with Classical Physics Theory.

⁷ Semi-classical means that the observer is sometimes in a classical referential, and sometimes in a quantum referential of observation.

⁸ Here, the prefix "super" does not mean superior conductivity tending to infinity, but rather, conductivity in a perpetual mode: without losses over a long period of time, which tends to infinity.

⁹ Here, suddenly means that the intensity of the resistance became very small, and consequently, it was a little sensitive to the experimental setup at that time.

bobbins, with normal conductivity, provided by an external source, was made up of spirals of copper wire and was immersed inside a single-walled recipient containing liquid air. This bobbin had the function of calibrating the intensity of its current with the current of the superconducting bobbin of lead spirals. A compass was placed between the two bobbins. In this way, when the current intensities were equal in both bobbins, the compass needle would move to the north. A simplified top-down view of this experiment is found in Fig. 03. Specific details of this experimental arrangement and conclusions can be found in reference [[28], 1914].

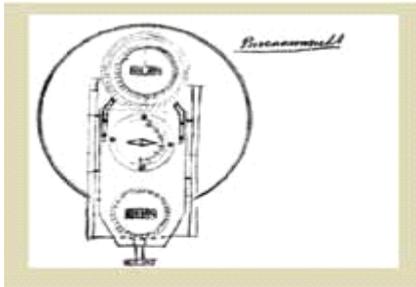


Fig.3. Simplified drawing of the experimental arrangement to detect a permanent current in a superconductor. The observer, Gerrit Flim¹⁰, made the drawing using a top-down directional view. The lead bobbin, superconducting, was placed to the west. The copper bobbin, normal conduction, was placed in the east. Both bobbins were the same size. A compass was placed between the bobbins to indicate current deflection in the bobbins [[28], 1914] (Fig. 03, Courtesy of the Boerhaave Museum).

In this experimental setup, an electric current was induced in the closed circuit of the superconducting bobbin by an induction magnet. This magnet was removed, then if any decrease in intensity occurred in the superconducting current in the bobbin, consequently, a deflection in the compass needle would occur. However, there was no decrease in the intensity of the 0.6 Ampere current that circulated in the superconducting lead bobbin immersed in liquid Helium.

Thus, in 1914, through an experiment, Kammerlingh Onnes managed to show permanent magnetic action in a superconductor, that is, a permanent current in a superconductor. In 1932, to confirm this reality, Gerrit Flim traveled with a Dewar flask to London. The flask contained a lead ring immersed in liquid helium, where a persistent current of 200 Ampere circulated in the ring. There was no decrease in current intensity during the entire voyage, as Kammerlingh Onnes had predicted through his experiments.

3.2 The Silsbee effect and the Meissner effect

The name of this subsection indicates that the objective in this subsection is to explain two important experimental phenomena of superconductivity: the Silsbee effect and the Meissner effect. In this direction, in 1916, Francis B. Silsbee (1889-1962) demonstrated through theory that:

“when an external magnetic field is applied to a superconducting state of a material, this material returns to its normal conducting state, if the intensity of the external magnetic field is equal to or greater than the critical value of the magnetic field of the superconducting state of this material. Likewise, the same will occur if an external electric current is applied to a superconducting state of a material” [[29], 1918]

¹⁰ Gerrit Flim was one of Heike Kammerlingh Onnes's assistants. This experiment was performed by Gerrit Flim only after Heike Kammerlingh Onnes's death.

This destruction property of the superconducting state became known as the Silsbee effect. In this case, two important facts have been proven today: **1)** the critical current intensity depends on the nature and geometry of the superconducting material, and **2)** a critical magnetic field greater than the magnetic field at the surface of the superconducting material cannot be induced.

In addition, concerning the action of the magnetic field, Walther Meissner (1882-1974) and Robert Ochsenfeld (1901-1993) developed experiments with the objective of understanding the effects of the external magnetic field on the superconducting state of a material, in particular, around its critical temperature or magnetic field. In 1933, the results of these experimental arrangements indicated that superconducting materials exhibited an interesting property. This property is the one contained in Fig. 04.

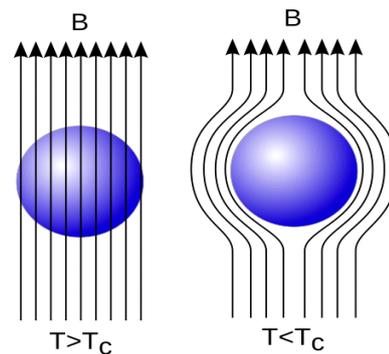


Fig.4. Action of an external magnetic field (B) on a material in the superconducting state. For a temperature (T) lower than the critical temperature (T_c), the intensity of the magnetic field B becomes lower than the intensity of the critical magnetic field induced by the current. Then, the magnetic field B bypasses this material, generating its suspension (levitation). For $T > T_c$, the intensity of the magnetic field B becomes greater than the intensity of the critical magnetic field induced by the current. Then, the magnetic field B passes through the superconducting material and destroys this superconducting state (the material returns to its normal conducting state), [[30], 1933].

This property of the superconducting state became known as the Meissner effect. This effect means that when an external magnetic field is applied to the superconducting material, the superconducting current reacts in the opposite direction and induces a magnetic field inside the superconducting state, which prevents the penetration of the external field. One consequence of this effect is that the superconducting object becomes suspended (**levitated**) by the action of the magnetic field. In this sense, in Fig. 04, on the right side, there is a simple visual representation of levitation.

In what follows, the next subsections will be discussing and explaining the first restricted theories developed in an attempt to explain the superconducting state behavior of a material.

3.3 Phenomenological Theory through Thermodynamics to better understand the Meissner effect

Cornelis Jacobus Gorter (1907-1980) and Handrik Brugt Gerhard Casimir (1909-2000) carried out theoretical studies on the Meissner effect to better understand its fundamentals and consequences. They realized that the temperature T , the external magnetic field \vec{H} and the magnetic field induced \vec{B} inside the material, in theory, could characterize the difference

between a superconducting state and a normal conducting state of the same material.

Both knew that the Helmholtz free energy function of a thermodynamic state contained T , \vec{H} and \vec{B} . Therefore, this function was expected to inform whether a material was in a superconducting state. In this direction, in 1934, both published an article [[31], 1934], where they used a phenomenological method supported by Thermodynamic Theory to better explain the causes and consequences of the Meissner effect.

In what follows, is a summary of this theory for the observant reader to save effort in understanding and time in reading the original article. In this sense, with the use of temperature and magnetic fields, the First Law of Thermodynamics becomes

$$dU = T \cdot dS + (1/4\pi) \cdot \vec{H} \cdot d\vec{B}, \tag{eq. 3.3.1}$$

where “d” represents a derivation differential, each point indicates a scalar operation and the magnetic fields are assumed in their vector forms. Thus, an energy differential (dU) is obtained as a function of the temperature (T), the external (\vec{H}) and induced (\vec{B}) magnetic fields and the entropy (S) of the system under analysis. Here, the induced magnetic field, the external magnetic field and the magnetization (\vec{M}) of the system under analysis, all in their vector forms, can be related through Ampère's Law, that is,

$$\vec{B} = \mu_0 \cdot (\vec{H} + \vec{M}), \tag{eq. 3.3.2}$$

where μ_0 is the vacuum permeability.

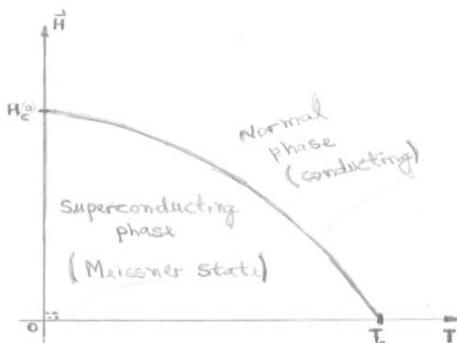


Fig. 5. Behavior of the superconducting material (metal) in relation to the external magnetic field (\vec{H}) versus the temperature (T) to which it is being subjected. In the graph, T_C represents the critical limiting temperature of the superconducting phase with the normal conducting phase. The behavior presented in this graph is characteristic of First Group Superconductors (Type I), which is being discussed in subsection 4.2 of this article

Now, a system of four equations in the four hidden variables T , \vec{H} , \vec{B} and S is obtained by specifying the Helmholtz¹¹ and Gibbs¹² free energy functions of the system under analysis as a function of temperature (T) and the magnetic fields \vec{H} and \vec{B} .

¹¹ Helmholtz free energy is a function that measures the portion of the internal energy of a system that can be used in the form of work and is represented by the function $F=U - T \cdot S$, where U is the internal energy and S is the entropy of the system under analysis.

¹² Gibbs free energy is a function that measures the total internal energy of a system available to perform useful work and is represented by the difference $G=G_f - G_i$, where G_f and G_i are, respectively, the final and initial Gibbs free energy of the system under analysis

From these equations, the intensity of the critical external magnetic field (H_C) is discovered and becomes dependent on the temperature of the superconducting state of the metal. In this sense, the graph in Fig. 05 shows this dependence in the formation of the Meissner effect.

As shown in Fig. 05, the observant reader realizes that the critical external magnetic field (H_C) occurs at the limit of absolute zero temperature (0 K). Furthermore, the observer sees that the relationship between the field and the temperature can be approximated by a parabolic behavior of the form

$$H(T) = H_c(0) \cdot [1 - T/T_c]^2, \tag{eq. 3.3.3}$$

where, if the intensity of the external magnetic field remains lower than this critical intensity, then the superconducting state is maintained. However, if the intensity of the external magnetic field remains greater than this critical magnetic field intensity, the superconducting state is destroyed and the material returns to its normal state of conduction. In addition, the interesting particular case occurs when $H=B_C=H_C$. In this case, the material becomes completely suspended (levitation).

Cornelis Jacobus Gorter and Handrik Brugt Gerhard Casimir, in this phenomenological theory, also demonstrated another interesting property of a superconducting material, which is the following:

$$G_N(T) - G_S(T) = \mu_0 \cdot H_C^2 / (8 \cdot \pi), \tag{eq. 3.3.4}$$

where, μ_0 represents the permeability of vacuum and H_C represents the critical magnetic field intensity. In verbal language, eq. 3.3.4 informs that the Gibbs free energy of the normal conducting state (G_N) is greater than the Gibbs free energy of the superconducting state (G_S). Therefore, the observer realizes that, when the material changes from the normal conducting phase to the superconducting phase, the Gibbs free energy becomes minimized by a difference of

$$\mu_0 \cdot H_C^2 / (8 \cdot \pi)$$

between the two. Furthermore, the observer sees that the Gibbs free energy of a superconducting state of a material has a dependence on the critical magnetic field with a quadratic exponent, while the Gibbs free energy of the normal conduction state of the same material is independent of the magnetic field. Consequently, for intense external magnetic fields ($H > H_C$), the superconducting state is destroyed and the material returns to its normal state of conduction.

In what follows, one more restrictive theory is discussed and analyzed in an attempt to explain the superconducting behavior of a material.

3.4 The London brothers' phenomenological theory to explain the Meissner effect and superconductivity

The Meissner effect was not satisfactorily explained until 1935. In this sense, Fritz London (1900-1954) and his brother Heinz London (1907-1970) knew that Ohm's Law and the Drude-Sommerfeld Model could not explain the superconducting state because in a superconductor, a surface

current manifests without the necessity of applying a potential difference ($V - V_0$) to the material.

Then, after extensive scientific research, in 1935, both brothers published an article [[32], 1935], where they demonstrated the existence of a second-order differential equation involving the magnetic field. The solution to this equation contained a parameter that represented a penetration coefficient (δ) of the magnetic field inside the superconducting material.

In the next paragraphs,, the demonstration of this equation and its solution are summarized, thus the observant reader will save time and effort in understanding the content of the original article. In this direction, consider Maxwell's Equation of electromagnetism (Ampère's Law),

$$\nabla \times \left(\frac{\partial \vec{H}}{\partial t} \right) = \frac{\epsilon}{c} \cdot \frac{\partial^2 \vec{E}}{\partial t^2} + \frac{4\pi}{c} \cdot \frac{\partial \vec{J}}{\partial t}, \tag{eq. 3.4.1}$$

where, ϵ is the dielectric constant and c is the speed of light in vacuum, \vec{H} , \vec{E} and \vec{J} are the external magnetic field, the electric field and the electric current density, respectively, always taken in their vector forms. Now, the observer should remember that the derivative of current density as a function of time, t , is $d\vec{J}/dt = (n \cdot q^2/m) \cdot \vec{E}$. Then the rotational operation of the rotational of the magnetic field can be performed and eq. 3.4.1 becomes

$$\nabla \times \left[\nabla \times \left(\frac{\partial \vec{H}}{\partial t} \right) \right] = \left[\frac{\epsilon}{c} \cdot \left(\frac{\partial^2}{\partial t^2} \right) + \frac{4\pi}{c} \cdot \frac{n \cdot q^2}{m} \right] \cdot \nabla \times \vec{E}. \tag{eq. 3.4.2}$$

Continuing, in this eq. 3.4.2, Faraday's Law of Induction can be used, $\nabla \times \vec{E} = -(1/c) \cdot (\partial \vec{H} / \partial t)$, which transforms the rotational operation of the rotational of the magnetic field into the physical-mathematical form

$$\frac{\partial}{\partial t} \left[\nabla \times (\nabla \times \vec{H}) + \left[\frac{\epsilon}{c^2} \cdot \left(\frac{\partial^2}{\partial t^2} \right) + 4\pi \cdot n \cdot \frac{q^2}{m \cdot c^2} \right] \cdot \vec{H} \right] = 0, \tag{eq. 3.4.3}$$

where q , m , n are respectively the charge, mass and electron density and the following algebraic artifice can be made $4\pi \cdot n \cdot q^2 / (m \cdot c^2) = 1/\delta^2$. At this point, the observant reader should note that Differential Equation 3.4.3 presents a temporal partial derivative operation. However, the Meissner effect of superconductivity does not involve time dependence, but instead involves temperature and magnetic field. Then the operation of the rotation of the rotation of the magnetic field becomes,

$$\nabla \times (\nabla \times \vec{H}) + \frac{1}{\delta^2} \cdot \vec{H} = 0, \tag{eq. 3.4.4}$$

where the following property of the rotational operation can be used: $\nabla \times (\nabla \times \vec{H}) = \nabla \cdot (\nabla \cdot \vec{H}) - \nabla^2 \vec{H}$ (In this property, the dot (\cdot) indicates the divergence operation of a vector). Thus, once this operation is done, this differential equation 3.4.4, independent of time, takes the physical-mathematical form.

$$\nabla^2 \vec{H} - \frac{1}{\delta^2} \cdot \vec{H} = 0. \tag{eq. 3.4.5}$$

This eq. 3.4.5 is the London Brothers Equation and presents spatial dependence (x, y, z) only. Now, for simplicity, the one-dimensional case (z -coordinates) can be assumed. In this case, this differential equation 3.4.5 takes the simplest form

$$\frac{\partial^2 \vec{H}}{\partial z^2} = \frac{1}{\delta^2} \cdot \vec{H}, \tag{eq. 3.4.6}$$

whose solution is a decendent exponential function:

$$\vec{H} = \vec{H}_i \cdot e^{-\frac{z}{\delta}} \tag{eq. 3.4.7}$$

This way, from Eq. 3.4.7, the observant reader can conclude that, in the Meissner effect, if the intensity of the external magnetic field (\vec{H}) is greater than the intensity of the critical magnetic field (\vec{H}_c), then the external magnetic field penetrates the interior of the superconducting material, but undergoes an exponential decay, $e^{-z/\delta}$ where the factor δ represents the quantitative coefficient of magnetic field penetration into the superconductor interior. In this sense, Fig. 06 contains a graphical representation of this effect.

In the next subsection, one of the most well known restrictive theories of superconductivity is discussed and analyzed: the BCS Theory.

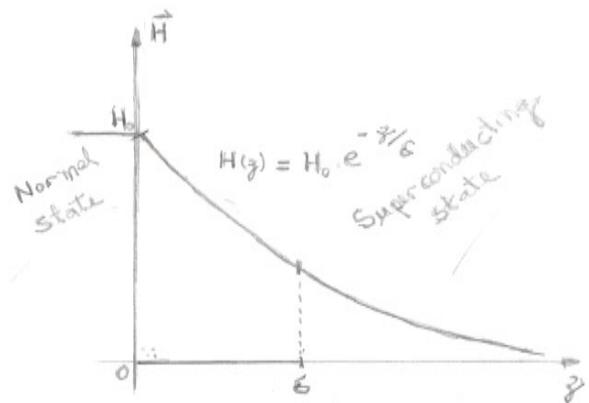


Fig.6. Behavior of the penetration of the external magnetic field ($H > H_C$) into the superconducting state of a material. The variable δ represents the penetration length of the London brothers in units of length. This behavior occurs mainly in First Group Superconductors (Type I), which are being discussed in subsection 4.2 of this article

3.5 The BCS Theory to explain superconductivity

By 1950, advances in Modern Physics and Quantum Mechanics had already made possible understanding many of the problems related to the interior of condensed matter, which Classical Physics was unable to contribute to solving. With this new knowledge, Emanuel Maxwell (1920-1972) wished to understand the meaning of the transition temperature, T_C , of a superconductor. Then, he performed many experiments with superconducting isotopes, which allowed him to conclude the following:

“The transition temperature is inversely proportional to the square root of the mass. Therefore, isotopes with greater mass have lower transition temperatures.” [[33], 1950]

Still in relation to isotopes, in the search for a theory that involved only physical definitions of the interior of matter¹³, Hebert Fröhlich (1905-1991) attempted to describe, through analysis of the isotope effect [[34], 1950], the behavior of electrons as a Degenerate Fermi Gas System and used the Perturbation Method to explain the effects observed in superconductivity. But he had difficulty generating an attractive interaction between electrons, which would cancel out the repulsive Coulomb Interaction. The difficulty arose because the crystalline lattice of the material did not adjust to isotopes of greater mass, given that their transition energies should be lower as a function of critical temperature¹⁴.

Also, in this quest to develop a theory for superconductivity, John Bardeen (1908–1991) knew that he had to create a system with a foundation in Field Theory [[35], 1950]. Then, he and Leon Neil Cooper (1930–2024) united to develop this theory together. In this direction, Cooper had demonstrated that:

“Independent of how small (weak) a bond between two electrons is, the Fermi Sea becomes unstable to form at least one bound electron pair” [[36], 1956]

Then, in this sense, a theory was developed by both as follows: in principle, since superconductivity was known to occur at absolute zero temperature, then they proposed a simple system in which two electrons, as a single particle, which do not interact with the Fermi Sea, move under this temperature. A simplified representation of the movement of this single particle is found in Fig. 07. Now, a wave function was needed to determine, the one that described the motion of this single particle, which was called the Cooper pair.

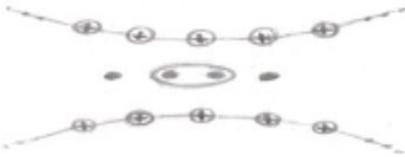


Fig. 7. Cooper pair in motion in a crystalline lattice of condensed matter, a small piece of the lattice. In this model, one electron distorts the lattice. This distortion attracts the other electron, and the two electrons together form a Cooper pair

In this way, in the search for the wave function, through the scientific works of Félix Bloch (1905-1983), John Bardeen and Leon Neil Cooper knew that the lowest energy occurred when the total quantity of movement of this pair of electrons was equal to zero. Consequently, the quantities of movement of the two electrons should be equal in module, but in opposite directions. Here, too, another scientific argument can be used to obtain the same conclusion, namely, that a non-zero total movement quantity means an electronic current initially, but without the presence of an electric field, which is a contradiction to the system proposed by both. By this means, then, the wave function should take the physical-mathematical form:

$$\psi_0(\vec{k}, \vec{r}_1, \vec{r}_2) = \sum_{\vec{k}} g_{\vec{k}} \cdot e^{i\vec{k} \cdot \vec{r}_1} \cdot e^{-i\vec{k} \cdot \vec{r}_2} = \sum_{\vec{k}} g_{\vec{k}} \cdot e^{i\vec{k} \cdot (\vec{r}_1 - \vec{r}_2)} = \sum_{\vec{k}} g_{\vec{k}} e^{i\vec{k} \cdot \vec{r}} \tag{eq. 3.5.1}$$

¹³ For example: charge, mass, electron energy, and so on.
¹⁴ Hebert Fröhlich had discovered that the vibration frequency of a lattice of condensed matter was proportional to the inverse square root of mass ($f = 1/\sqrt{M2}$) and this vibration frequency established a control link over the quantum of energy

where, for simplicity, an integral sum with respect to the coordinate r has been omitted, but which will be considered in another convenient part later.

Now, the observant reader realizes that exponential functions can be transformed into sums of cosine and sine using Euler's formula $e^{i \cdot x} = \cos(x) + i \cdot \sin(x)$. Additionally, the observer should remember that the spin relation must be imposed considering the asymmetry of the complete wave function, that is, $\psi_0(\vec{k}, \vec{r}) = -\psi_0(\vec{k}, \vec{r})$. These procedures being done in eq. 3.5.1, two partial wave functions are obtained, one containing the simplet spin function and the other containing the triplet spin function, which are respectively,

$$\sum_{\vec{k} > \vec{k}_f} [\tilde{E} - 2 \cdot \epsilon_{\vec{k}}] \cdot g_{\vec{k}} \cdot e^{i\vec{k} \cdot \vec{r}} = \sum_{\vec{k} > \vec{k}_f} V(\vec{r}) \cdot g_{\vec{k}} \cdot e^{i\vec{k} \cdot \vec{r}} \tag{eq. 3.5.2}$$

and

$$\sum_{\vec{k} < \vec{k}_f} (E - 2 \cdot \epsilon_{\vec{k}}) \cdot g_{\vec{k}} \cdot \int d\vec{r} e^{i\vec{k} \cdot \vec{r}} = \sum_{\vec{k} < \vec{k}_f} g_{\vec{k}} \cdot \int d\vec{r} V(\vec{r}) e^{i\vec{k} \cdot \vec{r}} \tag{eq. 3.5.3}$$

where $g_{\vec{k}}$ represents the probability amplitude of finding an electron pair with opposite movement quantity (momentum): ($\vec{p}_1 = \hbar \cdot \vec{k}$ e $\vec{p}_2 = -\hbar \cdot \vec{k}$). This probability amplitude becomes zero ($g_{\vec{k}} = 0$) for $|\vec{k}| < k_f = \sqrt{2} \cdot m \cdot E_f$. Here, in eq. 3.5.2 and eq. 3.5.3, α_1 represents the spin up and β_1 represents the spin down of electron 1 of the system, α_2 represents the spin up and β_2 represents the spin down of electron 2 of the same system under study.

The next action is to find out if both partial wave functions can be used to form the complete wave function and be used in the Schrödinger Equation ($H \cdot \psi_0 = E \cdot \psi_0$) to show a scientific reality. In this sense, only the partial wave function containing the **simplet spin** forms the complete wave function because the cosine function is even and, consequently, makes this function symmetric. Additionally, the simplet spin part generates an asymmetry in this same function, which is in the direction of the antisymmetry of the quantity of movement of the two electrons, which form the electronic pair (Cooper pair). Furthermore, the simplet spin function causes the total spin of the electron pair to be zero, which contributes to the use of the Schrödinger Equation (spin is not considered in this equation). The partial wave function containing the triplet spin part does not present these favorable properties for the asymmetric electron pair of the system under study. Therefore, this partial function must be rejected, it must not be part of the complete wave function. For example: the **triplet spin function** has a total spin equal to one (S=1), which makes it unfeasible to use in the Schrödinger's equation.

With this choice, the wave function containing the exponential form can be inserted into the Schrödinger's equation,

$$\left[\frac{\hbar^2 \vec{k}_1^2}{2 \cdot m} + \frac{\hbar^2 \vec{k}_2^2}{2 \cdot m} + V(\vec{r}_1, \vec{r}_2) \right] \cdot \psi_0(\vec{k}, \vec{r}_1, \vec{r}_2) = E \cdot \psi_0(\vec{k}, \vec{r}_1, \vec{r}_2) \tag{eq. 3.5.4}$$

to process sums only in the spaces of the Bloch vector \vec{k} and in the coordinate r. In this way, the following equation is obtained

$$\sum_{\vec{k} > \vec{k}_f} [E - 2 \cdot \mathcal{E}_{\vec{k}}] \cdot g_{\vec{k}} \cdot e^{i\vec{k} \cdot \vec{r}} = \sum_{\vec{k} > \vec{k}_f} V(\vec{r}) \cdot g_{\vec{k}} \cdot e^{i\vec{k} \cdot \vec{r}}, \quad \text{eq. 3.5.5}$$

where, $2 \cdot \mathcal{E}_{\vec{k}}$ represents the kinetic energy of the electron pair or dispersion energy as a function of the wave vector \vec{k}_f on the surface of the Fermi Sphere. Hence, $\mathcal{E}_{\vec{k}}$ represents the dispersion energy of an electron only on that surface. In Eq. 3.5.5 was also introduced that the spatial coordinate of the electron pair is $\vec{r} = \vec{r}_1 - \vec{r}_2$.

Now, the observant reader remembers that an integral sum in the spatial coordinate, $\vec{r} = \vec{r}_1 - \vec{r}_2$, of the electron pair had been omitted. Then, considering this integral sum, eq. 3.5.5 passes to the broader form:

$$\sum_{\vec{k} > \vec{k}_f} (E - 2 \cdot \mathcal{E}_{\vec{k}}) \cdot g_{\vec{k}} \cdot \int d\vec{r} e^{i\vec{k} \cdot \vec{r}} = \sum_{\vec{k} > \vec{k}_f} g_{\vec{k}} \cdot \int d\vec{r} V(\vec{r}) e^{i\vec{k} \cdot \vec{r}}, \quad \text{eq. 3.5.6}$$

At this point, the observer realizes that the integral sum on the left-hand side in eq. 3.5.6 contains a Dirac delta function. Hence, using the properties of this function, the integral takes on the value zero, for $\vec{r} \neq 0$, and the value 1, for $\vec{r} = 0$. Additionally, integration on the right-hand side of the same equation corresponds to a Fourier Transform in coordinate space \vec{r} to coordinate space \vec{k} . Hence, this integral sum can be considered as $\int d\vec{r} V(\vec{r}) \cdot e^{i\vec{k} \cdot \vec{r}} = V_{\vec{k}}$. With these operations, eq. 3.5.6 assumes the simplest form

$$\sum_{\vec{k} > \vec{k}_f} (E - 2 \cdot \mathcal{E}_{\vec{k}}) \cdot g_{\vec{k}} = \sum_{\vec{k} > \vec{k}_f} g_{\vec{k}} \cdot V_{\vec{k}}, \quad \text{eq. 3.5.7}$$

While the temperature is $T=0K$, both sides of Eq. 3.5.7 can be divided by the quantity $\sum_{\vec{k} > \vec{k}_f} g_{\vec{k}}$. This done, the following equation is obtained

$$1 = \sum_{\vec{k} > \vec{k}_f} \frac{V_{\vec{k}}}{(E - 2 \cdot \mathcal{E}_{\vec{k}})}, \quad \text{eq. 3.5.8}$$

however, in space \vec{k} , there are many possibilities for binding potential $V_{\vec{k}}$. But, Leon Neil Cooper assumed that this potential is constant with value $V_{\vec{k}} = -V$. By this means, eq. 3.5.8 becomes

$$\frac{1}{V} = \sum_{\vec{k} > \vec{k}_f} \frac{1}{(2 \cdot \mathcal{E}_{\vec{k}} - E)} \quad \text{eq. 3.5.9}$$

In this eq. 3.5.9, the observer remembers that the summation over the wave vector \vec{k} must be done in the space of the Fermi Sphere. Therefore, this summation must be transformed into an integral sum, where it must assume how many unoccupied points \vec{k} fit into a unit of volume of this Sphere. This procedure is a simple definition of the density of states \vec{k} . In this case, the Bloch wave vector can be given by $\vec{k} = (2 \cdot \pi / L) \cdot (n_x, n_y, n_z)$. These calculations make evident that only the symbol of the summation becomes an integral sum, and like this

$$\sum_{\vec{k} > \vec{k}_f} \rightarrow \frac{Vol}{(2 \cdot \pi)^3} \cdot \int d\vec{k}$$

which substituted into eq. 3.5.9, this eq. 3.5.9 takes the form

$$\frac{1}{V} = \frac{Vol}{(2 \cdot \pi)^3} \cdot \int d\vec{k} \frac{1}{(2 \cdot \mathcal{E}_{\vec{k}} - E)} \quad \text{eq. 3.5.10}$$

This integral sum in the wave vector \vec{k} , in eq. 3.5.10, can be resolved in spherical coordinates (θ, φ, k') of the system under study (in the Fermi Sphere). By doing so, the observant reader obtains that

$$\frac{1}{V} = \int \frac{Vol}{(2 \cdot \pi)^3} \cdot 4 \cdot \pi \cdot k^2 \frac{1}{(2 \cdot \mathcal{E}_{\vec{k}} - E)} dk \quad \text{eq. 3.5.11}$$

where only the two integral sums in the spherical coordinates θ and φ were resolved. Continuing the calculations, the integral sum in the one-dimensional wave vector k can be exchanged for an integral sum in the kinetic energy, which is also called dispersion energy. To do this, simply change the variable k to $\mathcal{E}_{\vec{k}}$ using that $\mathcal{E}_{\vec{k}} = 1 \cdot k^2 / (2 \cdot m)$, where, for simplicity, the constant \hbar was set identically to 1. The outcome of this procedure is that

$$\frac{1}{V} = \int \frac{Vol}{2 \cdot \pi^2} \cdot m \cdot \sqrt{2 \cdot \mathcal{E}_{\vec{k}} \cdot m} \cdot \frac{1}{(2 \cdot \mathcal{E}_{\vec{k}} - E)} d\mathcal{E}_{\vec{k}} \quad \text{eq. 3.5.12}$$

Now, the integral sum, in eq. 3.5.12, must be solved considering the lower limit of integration as being on the surface of the Fermi Sphere, that is, \vec{k}_f or E_f . The upper limit of integration must be considered in the vicinity of this surface, that is, $\vec{k}_f + d\vec{k}$ or $E_f + d\mathcal{E}_{\vec{k}}$. By the lower integration limit, at the surface of the Fermi Sphere, the factor

$$\frac{Vol}{2 \cdot \pi^2} \cdot m \cdot \sqrt{2 \cdot \mathcal{E}_{\vec{k}} \cdot m},$$

in eq. 3.5.12, is the density of states or points or dispersion energy on the surface of the Fermi Sphere, when the temperature $T = 0 K$, this density is constant and called $N(0, \vec{k}_f)$ or $N(0, \mathcal{E}_{k_f})$ or, simply, $N(0)$. Then, this factor can come out of the integral sum and be placed by multiplying this integral sum. Consequently, eq. 3.5.12 takes on its simplest form

$$\frac{1}{V \cdot N(0)} = \int \frac{1}{(2 \cdot \mathcal{E}_{\vec{k}} - E)} d\mathcal{E}_{\vec{k}} \quad \text{eq. 3.5.13}$$

At this point, the solution to the integral sum in this eq. 3.5.13 can be obtained from an integral solution table. But, the solution of this integral sum can be obtained by changing the variable $(2 \cdot \mathcal{E}_{\vec{k}} - E)$ to x , for example. Thus proceeding and knowing that the lower and upper integration limits, respectively, are E_f and $E_f + \hbar \cdot \omega_D$, the observant reader obtains that

$$\frac{1}{V \cdot N(0)} = \frac{1}{2} \cdot \log_{10} \left[\frac{2 \cdot E_f - E + 2 \cdot \hbar \cdot \omega_D}{2 \cdot E_f - E} \right], \quad \text{eq. 3.5.14}$$

where the exponential function can be applied to both sides and, finally, through algebraic manipulations, the energy variable is obtained in the form expressed by

$$E - 2.E_f = \frac{-2.\hbar.\omega_D}{e^{\frac{2}{V.N(0)}} - 1} \quad \text{eq. 3.5.15}$$

Here, the observant reader may recall that for most conventional superconductors, the exponent $V.N(0) < 0,3$, as stated in the reference [[37], 1957]. For this reason, a Taylor expansion can be done in the denominator $(e^{\frac{2}{V.N(0)}} - 1)$ from eq. 3.5.15. From this expansion, an equation in the weak coupling condition is obtained in the form

$$E - 2.E_f = -2.\hbar.\omega_D . e^{-\frac{2}{V.N(0)}} \quad \text{eq. 3.5.16}$$

Finally, from this result, eq. 3.5.16 and eq. 3.5.15, the following can be concluded: **1)** if, $2.E_f > E$, then a bound state of electrons with a wave vector $\vec{k} > \vec{k}_f$, just above the surface of the Fermi Sphere or just above the Fermi energy E_f , is obtained, even if this bound state has a weak binding intensity; **2)** This energy difference indicates that a finite quantity of energy is necessary to break the bond between the pair of electrons, which implies that there is an energy gap for single particle excitation, in contrast to the normal state of conduction, where the state of an electron can be changed by adding any small quantity of energy; **3)** and this energy gap depends on temperature, disappears at the transition temperature, T_c , and is greater at lower temperatures.

In 1956, Michael Tinkham (1928–2010) and Rolfe Eldridge Glover III (1924–1987) placed a superconducting material under infrared radiation. Using a spectrometer, they realized that energy was absorbed by the material in discrete quantities, one quantum at a time, in the continuous distribution of energy levels. Thus, the existence of an Energy Gap was characterized [[38], 1957], as had been predicted in the restricted BCS Theory developed by John Bardeen, Leon Neil Cooper and John Robert Schrieffer. With this result, Part 2 is closed and Part 3 of this article is initiated.

4. Part 3

The Ginzburg-Landau Theory, Abrikosov's solution to the Ginzburg-Landau Differential Equations and the tunneling of Cooper pairs between superconductors proposed by Josephson are discussed and analyzed in this Part 3. Thus, the objective in this Part 3 is to demystify misconceptions and provide greater understanding of these issues.

4.1 Ginzburg-Landau's proposal to explain superconductivity

In 1950, Vitaly Lazarevich Ginzburg (1916–2009) and Lev Davidovich Landau (1908–1968) proposed a theory to explain superconductivity [[39], 1950]. In this theory, Landau's theory of second-order phase transitions and Schrödinger's wave equation are combined to generate new equations.

They began the theory by proposing the following: **1)** a superconductor contains a density of superelectrons, n_s and a normal electron density, $n - n_s$, where n represents the total electron density in the metal and n_s is not necessarily homogeneous in space; **2)** the charge carriers in the superconducting state are seen by the observer as a quantum

fluid, which evolves by means of a macroscopic wave function $\psi(\vec{r})$, which represents the superconducting **order parameter**; **3)** the Helmholtz free energy density of a superconductor, close to the superconducting transition, can be expressed in terms of the complex order parameter,

$$\psi(\vec{r}) = |\psi(\vec{r})| . e^{i.\phi(\vec{r})} \quad \text{eq. 4.1.1}$$

such that, represents the local electron density in the superconducting state, and yet, **4)** by conclusion, $\psi(\vec{r}) \neq 0$ is assumed in the superconducting state, however, $\psi(\vec{r}) = 0$ is assumed in the normal state of conduction. In other words,

$$\text{if, } T > T_c, \text{ then } \psi(\vec{r}) = 0 \text{ and if, } T < T_c, \text{ then } \psi(\vec{r}) \neq 0$$

Continuing, Vitaly Lazarevich Ginzburg and Lev Davidovich Landau realized that the Thermodynamic description of this theoretical Model could be obtained by expanding the Helmholtz free energy of the superconducting state in powers of $(\psi(\vec{r}))^2$ and the potential vector \vec{A} . In this expansion, in the vicinity of the critical temperature, T_c , as in Landau's second-order phase transition, only the first terms of the expansion can be considered. From this calculus, the Helmholtz free energy density comes to present a physical-mathematical form of Field Theory, that is (**eq. 4.1.2**),

$$f_s(\psi, \vec{A}) = f_n + \alpha . |\psi(\vec{r})|^2 + \frac{1}{2} . \beta . |\psi(\vec{r})|^4 + \frac{1}{2.m} . \left[-i.h.\nabla - \frac{e}{c} . \vec{A}(\vec{r}) \right] . |\psi(\vec{r})|^2 + \frac{\vec{B}^2}{8\pi} = 0,$$

where, f_s and f_n , respectively, are the Helmholtz free energy density of the superconducting state and the normal conduction state, m represents the effective mass, e represents the effective charge, $2.e$, where e is the charge of an electron, \vec{A} represents the potential magnetic vector with $\nabla \times \vec{A} = \vec{B}$, where \vec{B} is the magnetic field induced by the \vec{A} and, finally, α and β are coefficients. They depend on temperature and can be obtained from the phenomenological conditions of the system, for example, equilibrium, expansion, and so on. In general, α and β have the following properties:

$$\text{if, } \alpha(T) > 0, \text{ then } T > T_c, \text{ if, } \alpha(T) < 0, \text{ then } T < T_c \text{ and } \alpha(T=0) = 0 \text{ and } \beta(T=0) > 0$$

Now, the observant reader realizes that each term in the energy density expansion, eq. 4.1.2, has a scientific meaning related to the superconducting state or the normal conduction state. In this direction, the first term represents the energy of the normal conduction state. The condensation energy of the superconducting state is represented by the second and third terms of the expansion. The fourth term represents the kinetic energy of charge carriers in the superconducting state. The last term, the fifth, corresponds to the increase in energy sufficient to maintain the flow outside the superconductor. From everything exposed, the equation for ψ can be obtained from the Total free energy of the system, that is,

$$F_s(\psi, \vec{A}) = \int f_s(\psi, \vec{A}) d^3\vec{r}, \quad \text{eq. 4.1.3}$$

where replacing eq. 4.1.2, **eq. 4.1.4** is obtained:

$$F_s(\psi, \vec{A}) = f_n + \int d^3\vec{r} \left[\alpha . |\psi(\vec{r})|^2 + \frac{1}{2} . \beta . |\psi(\vec{r})|^4 + \frac{1}{2.m} . \left[-i.h.\nabla - \frac{e}{c} . \vec{A}(\vec{r}) \right] . |\psi(\vec{r})|^2 + \frac{\vec{B}^2}{8\pi} \right]$$

In eq. 4.1.4, if the energy is infinitely small, the observant reader can perform the calculation or variation Method with respect to ψ and $\partial\psi$. Thus, eq. 4.1.5 is obtained, that is

$$\alpha \cdot \psi + \frac{1}{2m} \cdot \left[\frac{\hbar}{i} \cdot \nabla - \frac{e}{c} \cdot \vec{A}(\vec{r}) \right]^2 \cdot \beta |\psi(\vec{r})|^2 \cdot \psi + \frac{1}{2m} \cdot \left[\frac{\hbar}{i} \cdot \nabla - \frac{e}{c} \cdot \vec{A}(\vec{r}) \right]^2 \cdot \psi = 0,$$

at this point, the observer realizes that this differential equation is analogous to Schrödinger's equation in Quantum Mechanics for a free particle, but it is a little different from this one because it contains a nonlinear term.

Likewise, the equation for the vector potential \vec{A} can be obtained if the observer assumes that $\text{div}(\vec{A})=0$ and to vary the Total free energy with respect to the vector potential. In this way, the following equation is obtained

$$\nabla^2 \vec{A} + \frac{4\pi}{c} \cdot \vec{J} = 0 \quad \text{eq. 4.1.6}$$

with

$$\vec{J} = \frac{e \cdot \hbar}{2m} \cdot [\hat{\psi} \cdot \nabla \psi - \psi \cdot \nabla \hat{\psi}] - \frac{e}{2m \cdot c} \cdot |\hat{\psi}| \cdot \vec{A}, \quad \text{eq. 4.1.7}$$

where $\hat{\psi}$ represents the conjugate of ψ and \vec{J} represents the supercurrent density similar to that of Quantum Mechanics.

Equations 4.1.5 and 4.1.6 are the Ginzburg-Landau equations. In the use of these equations, the necessary boundary conditions are introduced and the London brothers' theory is used to solve specific problems in finite samples. In this respect and others in the demonstration, the reference [[39], 1950] contains the paths of calculations for some of these more specific cases.

Moving forward, with eq. 4.1.5 and eq. 4.1.6, Ginzburg-Landau determined the surface energy of the plane boundary between the superconducting and normal conduction phases for the following two cases: **one**, when the magnetic field $\vec{H}=0$ and the **other**, when the magnetic field is independent of time. In both cases, they considered only one-dimensional space. During these calculations, spontaneously, by the theory itself, Ginzburg-Landau defined the following parameters of length scale variation: **1) the penetration length**, δ , which corresponds to that obtained in the London Brothers Theory for superconductivity, this δ has the meaning of providing the measurements, on a spatial scale, of the variations in the field and current in the penetration into the superconductor; **2) the coherence length**, ξ , which has the meaning of providing the measure, on the escaical scale, of the variation of the order parameter ψ ; and **3) the Ginzburg-Landau parameter**, \bar{K} , which is the reason given by the division of δ by ξ . This \bar{K} , proportionally, is related to measurements of the critical magnetic field, \vec{H}_c , and it has become of fundamental importance in the analysis of electronic states at the interface between the superconducting phase and the normal conduction phase.

With these parameters, Ginzburg-Landau managed to rewrite eq. 4.1.5 and eq. 4.1.6, in such a way that the case of an external magnetic field $\vec{H}_c=0$ became explicit within these

one-dimensional differential equations, mainly in the differential equation for ψ . Ginzburg-Landau solved these equations and found solutions for the order parameter ψ and for the vector potential \vec{A} , which explicitly became dependent on the Ginzburg-Landau parameter \bar{K} . Through this parameter \bar{K} , they analyzed what was happening on the surface of the interface between the superconducting phase and the normal conduction phase. Then, both concluded the following:

(1) for the external magnetic field $\vec{H}=0$ implies

$$H_{cb}^2 = \frac{4\pi \cdot \alpha^2}{\beta} = \frac{4\pi \cdot (T_c - T)^2}{\beta_c} \cdot \frac{d\alpha}{dT} \cdot c^2, \quad \text{eq. 4.1.8}$$

which corresponds to the critical external magnetic field for the Meissner effect to remain as displayed in Fig. 05, in subsection 3.3;

(2) for external magnetic field \vec{H} independent of time, the analysis was being done on the parameter of Ginzburg-Landau, \bar{K} , in this way:

$$\text{2a) if, } \bar{K}^2 = \frac{1}{2\pi} \cdot \left(\frac{m' \cdot c^2}{e \cdot \hbar} \right)^2 \cdot \beta = \frac{2 \cdot e^2}{\hbar^2 \cdot c^2} \cdot \vec{H}_{cb}^2 \cdot \delta_0^4 = 0 \quad \text{eq. 4.1.9}$$

implies that the order parameter $|\psi_0(\vec{r})|^2 = n_s$, which is constant, therefore, the Ginzburg-Landau differential equations show the possibility of determining only one critical external magnetic field intensity, \vec{H}_{cb} , and are transformed into the London brothers' equations for superconductivity, with the penetration length in second power, that is,

$$\delta^2 = \delta_0^2 = \frac{m' \cdot c^2}{4\pi \cdot e^2 \cdot |\psi_0(\vec{r})|^2}, \quad \text{eq. 4.1.10}$$

2b) for the limit case of $\bar{K} \rightarrow 0$,

the penetration length, δ , presents the quantity of penetration for a non-intense magnetic field, which is called a weak magnetic field, going into the surface of the superconductor;

2c) if, $\bar{K} > 0$,

the solution to the Ginzburg-Landau differential equations only exists for a certain critical external magnetic field \vec{H}_{c2} , the range of field intensities \vec{H}_{cb} becomes

$$\left(H_{cb} = \frac{1}{\sqrt{2}} \right) < H < H_{c2}, \quad \text{eq. 4.1.11}$$

where the superconducting states, which can exist mixed with the normal conduction states, are called metastable states, but the absolute minimum free energy is in the normal conduction phase trend, Fig. 08 provides a graphical representation of this case; and

2d) if, $\bar{K} \geq \frac{1}{\sqrt{2}}$,

an instability occurs in the normal conduction phase, the instability arises due to the formation of thin layers of the

superconducting phase, however, this analytical study did not come to a completely closed conclusion, remained questionable and open.

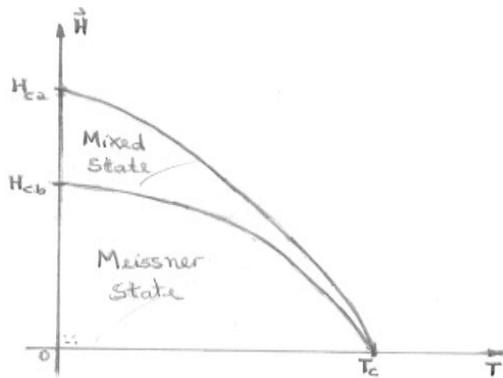


Fig.8. Presence of a second critical external magnetic field intensity ($\bar{H}_{c2} > \bar{H}_{cb}$), which confines mixed states between both intensities of the critical magnetic field. The critical magnetic field intensity \bar{H}_{cb} is the frontier of the superconducting state, where the Meissner effect occurs

Thus, with this restricted phenomenological theory supported by the London brothers' theory, other scientific researchers were able to solve the following problems: **1)** the effects of fields intense enough to change the density of superelectrons n_s or the local electron density in the superconducting state; **2)** problems involving the spatial variation of the electron density in the superconducting state, n_s ; and **3)** beyond to dealing with the intermediate state of some superconductors, where normal conduction states and superconducting states coexist in the presence of a second magnetic field of intensity a little greater than the critical magnetic field intensity \bar{H}_{cb} , in this case, $\bar{K} > 0$, but not greater than $1/\sqrt{2}$.

4.2 Abrikosov's solution to the Ginzburg-Landau differential equations

In continuation of subsection 4.1 of this article, for the Ginzburg-Landau parameter $\bar{K} > 1/\sqrt{2}$, Vitaly Lazarevich Ginzburg and Lev Davidovich Landau showed that superconductivity is permanent for magnetic fields that are intense compared to the critical magnetic field H_{cb} , for which equilibrium can exist between normal conducting states and superconducting states [[39], 1950]. However, in this case, the study was conducted for values of \bar{K} not much larger than $1/\sqrt{2}$, therefore, the problem of superconductivity in this case has not been conclusively closed.

Then, Alexei Alexeyevich Abrikosov (1928–2017) studied the article published by Vitaly Lazarevich Ginzburg and Lev Davidovich Landau on superconductivity. A few years later, in 1957, Abrikosov published an article about this study and questions about superconductivity [[40], 1957]. In this article, he solved¹⁵ the Ginzburg-Landau differential equations for the case of the parameter $\bar{K} > 1/\sqrt{2}$, which corresponds to a magnetic field intensity \bar{H} much greater than the critical external magnetic field \bar{H}_{cb} , which lies between the superconducting transition and the normal conduction phase. In this solution, Abrikosov concluded what follows:

- The Ginzburg-Landau theory of superconductivity leads to the assertion that there are two groups of superconductors;
- for the Ginzburg-Landau parameter $\bar{K} > 1/\sqrt{2}$, which corresponds to the intensity of magnetic fields \bar{H} less than or equal to the critical external magnetic field H_{cb} , superconductors that can withstand these magnetic field intensities can be called First Group Superconductors or Type I Superconductors, this group exhibits Meissner effect behavior, and the transition occurs in the first order, Fig. 05, in subsection 3.3, represents this First Group of superconductors;
- for the Ginzburg-Landau parameter $\bar{K} > 1/\sqrt{2}$, which corresponds to the intensity of magnetic fields \bar{H} much greater than the critical magnetic field intensity of the first order of transition H_{cb} , but less than the intensity of a third critical magnetic field limit H_{c3} , superconductors that can withstand these magnetic field intensities can be called Second Group Superconductors or Type II Superconductors, in this group, the second critical magnetic field limit H_{c2} , ($H_{c2} < H_{c3}$, but $H_{c2} > H_{cb}$), confines the mixed states, which Ginzburg-Landau predicted to exist in the second order of transition, however, for magnetic field intensities \bar{H} , with $H_{c2} < \bar{H} < H_{c3}$, surface superconductivity arises, Fig. 09 represents this Second Group of superconductors, and finally,

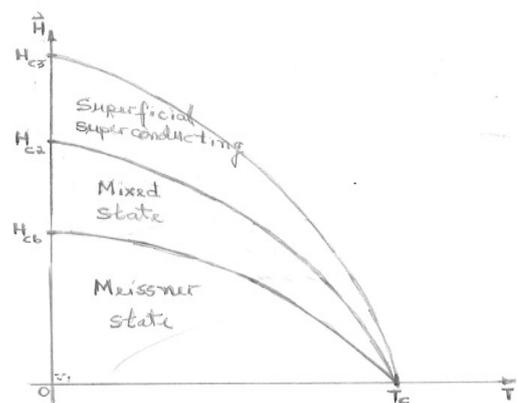


Fig.9. Superconducting states of a material as a function of critical magnetic field intensities H_{cb} , H_{c2} and H_{c3} . The behavior exhibited in this graph is that of Second Group Superconductors (Type II), which contains these three types of behavioral states, that is, the Meissner effect, mixed states, and superficial superconductivity, for the same critical temperature, T_c , supported by these states of the superconducting material, in general, metals.

- still for the Ginzburg-Landau parameter $\bar{K} > 1/\sqrt{2}$, in mixed states, which occur between the intensities of critical magnetic fields H_{cb} and H_{c2} , supercurrents exist circulating around regions not far from the frontier with the normal state of conduction and forming supercurrent vortices, where the magnetic field flux, Φ , decays in intensity characteristic penetration length δ , but each vortex of the supercurrent concentrates the maximum intensity of the Φ at the center of its vortex and quantized in value of

$$\Phi = \frac{h}{2e} = 2,0679 \times 10^{-15} \text{ T} \cdot \text{m}^2, \quad \text{eq. 4.2.1}$$

in this way, the magnetic field flux remains flowing in tube-shaped geometric regions, and since $2 = 0$ at the centers of the supercurrent vortices, then Energy Gap does not exist in these

¹⁵ He solved the differential equations by assuming a single direction (one-dimensional) and using convenient approximations from Field Theory.

centers, therefore, superconductors may not be perfect in their conduction of permanent current and energy (some losses may occur). As an illustration, Fig. 10 contains an image representing a supercurrent vortex.



Fig. 10 - Supercurrent vortices forming a vortex (swirl) in a mixed state. The observer realizes that the reference point for observing the vortex was made from top to bottom, and here, only one vortex was made, but there are lots of vortices in this state close to the normal conduction state. (This figure represents an illustration in a plane)

Onward, one more interesting superconducting behavior: Josephson tunneling and a new definition to characterize superconductivity is being discussed and analyzed in the next subsection.

4.3 Is tunneling of Cooper pairs possible between superconductors?

In the previous subsection, the solutions of the Ginzburg-Landau Equations given by Alexei Alexeyevich Abrikosov and others indicated that Type II Superconductors may not be perfect conductors with respect to the permanent current of Cooper pairs. Consequently, another universal characteristic, which represents the superconducting state for these superconductors, was necessary to find for these superconductors. For this reason, the following important question was raised: What is the universal characteristic that sustains the permanent current state in these Type II Superconductors?

For this question, in the referential of Quantum Mechanics Theory, one possible answer was to affirm that, for many particles, wave functions, ψ , exist in which the amplitude and phase maintain coherence over long distances of the size of the superconducting state scale (macroscopic limit). In this sense, wave-particle duality can be supported by Heisenberg's Uncertainty Principle, but instead of momentum and position in uncertainty, the phase ϕ and the number of particles N become the conjugate variables. As a result, Heisenberg's Uncertainty Principle can be represented by

$$\Delta N \cdot \Delta \phi \leq 1. \quad \text{eq. 4.3.1}$$

Thus, as the wave function with its phase and amplitude can characterize a superconducting state, in 1962, Brian David Josephson published an article [[41], 1962], in which, by theory, he sustained that Cooper pairs can cross two or more superconductors separated by a thin layer¹⁶. In other words,

¹⁶ The thin layer can be made of some non-superconducting material, for example, a normal conductive material or an insulator. With these two materials, respectively, a SNS junction or a SIS junction can be formed. Also, a SsS junction can be formed in which two superconductors touch only at a

Cooper pairs can obey the behavior of the Tunnel effect. With this proposal, the problem was to find the formula for the current I , which is transformed into a non-homogeneous differential equation with the angle ϕ (the phase) as the unknown variable. From there, he found the solution to this equation using the standard solution Method and making physical and mathematical considerations related to the construction of the problem under study. In this way, Brian David Josephson demonstrated that:

- (1) if $|I| \leq I_c$, where I_c is the critical current, then the general solution arises by taking the angle ϕ constant, this result means that the voltage at the Josephson junction is zero, thus, the current source can be adjusted between $-I_c$ and $+I_c$, without voltage appearing between the superconductors, this effect is called Josephson DC junction. and the ohmic current contribution in the differential equation is zero; and
- (2) if $|I| > I_c$, then the general solution becomes more complicated to obtain, but it arises by taking the angle ϕ dependent on the time variation t , thus, $\phi(t)$ takes on a phase meaning and not just an angle meaning, in this way, the temporal derivative of $\phi(t)$ indicates the natural emergence of a time-dependent voltage at the Josephson junction, that is,

$$V(t) = \frac{\hbar}{2e} \cdot \frac{d\phi(t)}{dt}, \quad \text{eq. 4.3.2}$$

where e is the elementary charge of the electron and

$$\hbar = \frac{h}{2\pi} \approx 6,58 \times 10^{-16} \text{ eVs}$$

However, the spontaneous appearance of a voltage $V(t)$ requires the determination of an average of this time-dependent voltage because, for an average voltage different from zero, a high oscillation frequency arises in the current at the Josephson junction. This case is called Josephson AC junction, and the average voltage is determined by

$$V = \frac{1}{T} \cdot \int_0^T V(t) dt = \frac{\hbar}{2e} \cdot \frac{2\pi}{T} \quad \text{eq. 4.3.3}$$

Here, the observant reader realizes that the proportionality constant $\hbar/2e$, which relates the average voltage V and the inverse of the period ($1/T$), which is the oscillation frequency, depends only on the elementary charge and Planck's constant \hbar . This constant $\hbar/2e$ does not depend on the characteristics of the superconductor or the experimental setup constructed nor the temperature or the exact intensity of the current. Therefore, this result makes evident that periods and frequencies can be measured with the highest precision using Josephson tunneling. For this reason, the relationship between time and voltage of Josephson junctions was used to construct a standard for the Volt unit. In this sense, the constant $\hbar/2e$ was calculated and established as the quantity

$$2,069 \times 10^{-15} \text{ V}_s = \frac{\hbar}{2e}, \quad \text{eq. 4.3.4}$$

that is, a frequency of 1 GHz corresponds to a voltage of approximately $2\mu\text{V}$.

thin edge. At the SNS junction, the non-superconducting layer can have a thickness of the order of 10 \AA and at the SIS junction, the non-superconducting layer can have a thickness of the order of 200 \AA .

Now, considering the current portion related to the capacitance of the Josephson junction and assuming many junctions in series, this proportionality constant can be determined more accurately. These calculations showed that

$$\frac{2 \cdot e}{h} = 2,4835979 \times 10^{-15} \frac{1}{s V_{90}}, \quad \text{eq. 4.3.5}$$

where, the quantity $2,4835979 \times 10^{-15}$ was chosen at this value in such a way that the new voltage unit remained as close as possible to the old voltage unit V.

Specific details of the calculations about Josephson tunneling can be found in reference [[41], 1962], for example. In terms of practical application, Josephson tunneling or Josephson junctions have great application potential. In this sense, for example, they can be used to build an instrument capable of detecting magnetic fields of the order of 10^{-15} Tesla, which are called magnetic fields that are too weak to be detected. Thus, the magnetic fields generated by the Earth and the human brain, which are respectively of the order of 10^{-6} Tesla and 10^{-13} Tesla [[42], 2013], could be measured with extreme accuracy.

To finish up this subsection, in 1963, Philip Warren Anderson (1923–2020) and John M. Rowell (1935–living) developed an experimental setup [[43], 1963], where they were able to verify the existence of Cooper pair tunneling, which had been proposed in theory by Brian David Josephson.

5. Part 4

This Part 4 is dedicated to answering the following actual question: Can a state of superconductivity have a transition temperature equal to Earth's ambient temperature and pressure?

In this direction, from the Second Group Superconductors and the new concept of superconductivity, which were discussed at the end of subsection 4.2 and in subsection 4.3 of this article, in principle, perhaps a state of superconductivity could have a transition temperature equal to Earth's ambient temperature and pressure. For this reason, Group II Superconductors are also called "High-temperature" Superconductors and the following questions have been awakened by scientific community: (1) How to make a superconductor that has a transition temperature and pressure at the same values as the Earth's ambient temperature and pressure? (2) Could this be obtained by forming chemical element alloys, chemical doping, or another method not yet known?

Thus, in this sense, in 1986, Johannes Georg Bednorz (1950–alive) and Karl Alex Müller (1927–2023) built a superconductor formed from a composite of barium oxide, lanthanum and copper, which became known as perovskite cuprate. The transition temperature this superconductor was of the order of **35 K**. Afterwards, they realized that replacing lanthanum by yttrium formed the YBCO superconductor, which presented a transition temperature of the order of **92 K** [[44], 1986]. In 1987, Paul Ching-Wu Chu and his group synthesized a ceramic, that became known as $\text{Yb}_2\text{Cu}_3\text{O}_{7-x}$, which exhibited a critical temperature of the order of **93 K** [[45], 1987].

With temperatures **above 90 K**, now, superconducting samples could be cooled with liquid Nitrogen, which has a boiling point at a temperature of 77 K. In addition, for various other superconductors, Fig. 11 shows a summary of superconducting materials developed between 1900 and 2015 and their critical temperatures, which are higher in more recently constructed superconducting materials.

From 1987 to 2024, various other superconducting materials were constructed, which presented transition temperatures much higher than 90 K. The observant reader can consult Fig. 11 to verify part of this fact. Among these, in 2004, Neil W. Ashcroft reminded that, since 1971, John Gilman had proposed hydrogen-abundant materials (hydrides) to generate high transition temperatures in a superconducting state [[46], 1971], [[47], 2004]. In this vein, in 2014, A. P. Drozdov, M. I. Eremets, and I. A. Troyan developed hydride superconductors at the Max-Planck Institute of Chemistry and Physics: sulfur hydride and hydrogen sulfide [[48], 2015]. The transition temperatures these superconductors were of around **190 K** and their resistance tending to zero, but when subjected to high pressure, pressure much higher than Earth's ambient pressure. More recently, still using the same technique, in 2019, Yanming Ma and his collaborators at Jilin University proposed that $\text{Li}_2\text{MgH}_{16}$ should have a transition temperature of the order of **473 K** (200°C), when submitted to pressure of 250 Gpa [[49], 2019]. This is enormous pressure, which is equivalent to the pressure at the center of the planet Earth. For this reason, this superconductor became unfeasible to build on a large scale for production and utility.

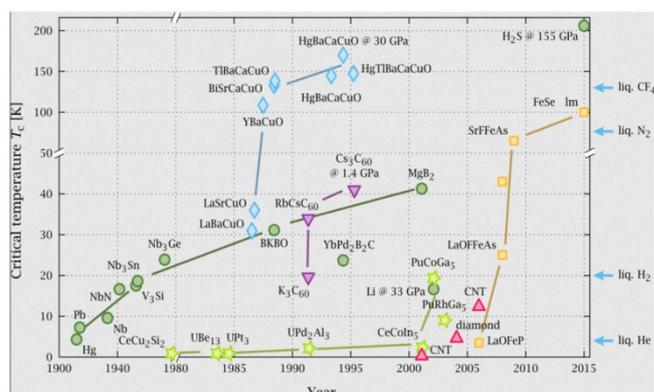


Fig. 11 - Increase in the transition temperature, T_C , as a function of superconductors developed from 1900 to 2015. The constructive search for a state of superconductivity originated within Earth's ambient temperature and pressure continues to this day. This superconductor has not yet been constructed. (This map is useful for everyone in the field of superconductivity)

Still in relation to the manufacture of superconductors using the high pressure method, in 2020, some articles published in this vein were immediately refuted by other groups of scientific researchers. Therefore, the authors of these articles had to retract these publications, for example, the retraction established in reference [[2], 2022] is one of these cases.

6. Conclusion

Finally, this scientific researcher ends this article with an extended conclusion, which is divided into three subsections, as soon follows.

6.1. The evolution of the concept or definition of superconductivity

The first concept or definition of superconductivity emerged around 1900, when Heike Kammerlingh Onnes and his collaborators showed through experimental measurements that the resistance of a pure metal tends to zero as its temperature approaches absolute zero (0 K). In this case, an induced electric current appears in the metal, which remains at the same intensity for an indefinite period of time (**perpetual mode**).

This result refuted the proposal that the electronic state becomes frozen as its temperature tends toward absolute zero and, consequently, the resistance of this metal tends to infinity. This refuted concept had been defended by Lord Kelvin and others through a view referential given by Classical Physical Theory. But the first concept or definition of what superconductivity is, essentially, was obtained using the observation referential given by Quantum Mechanics Theory, which was in the substantiation procedure (**Quantum Principles**).

The second concept or definition of superconductivity arose from the interest in obtaining a superconducting material in which the transition temperature and pressure were those of the Earth's ambient conditions. These superconductors are Second Group Superconductors (Type II), also known as "High Temperature" Superconductors. In this case, in the superconducting state, the concept of zero resistance can cease to exist. But an electric current or Cooper pair current of the same intensity (constant) is still possible to secure for an indefinite period of time (**perpetual mode**) discounting the loss due to resistance, for example, dissipation, and so on.

In these Type II Superconductors, because energy can be lost, even if minimal, zero resistance ceases to be the characteristic parameter of the superconducting state. Now, since this problem deals with many bodies or many particles, then, from the Quantum Mechanics referential, the Wave Function, which carries a **phase** and an **amplitude**, was adopted to represent the characteristic parameter of the superconducting state of a material. In this way, the **phase** and **amplitude** can indicate the coherence of this state over long distances of the size of the superconductor scale.

6.2 - The evolution of the theory to describe the state of superconductivity

Up to the present moment, some short-range theories were elaborated to describe the superconducting state of a material. All these theories were developed through experimental studies of a vast quantity of materials that exhibited the superconducting state. For example: **(1)** BCS Theory, which was the first theory described only with specific mathematical physics for the interior of matter, but this theory only involved the ground state ($T=0$ K), in this case, theory predicted that this state is separated from the excited state by an Energy Gap, this theory is described in subsection 3.5 of this article, and **(2)** Ginzburg-Landau Theory, in which differential equations are constructed and supported by the second-order transition Landau Theory, these equations must be solved for within the superconducting material with the help of the Londo brothers' equations among others; this theory was complemented by Alexei Alexeyevich Abrikosov, in 1957, when he solved these

differential equations for within the superconducting material and obtained solutions different from those of Ginzburg-Landau and, among other conclusions, he defended that there are two different groups of superconductors: First Group Superconductors, also called Type I Superconductors, and the Second Group Superconductors, also called Type II or "High Temperature" Superconductors, an introduction to these theories is found in subsections 4.1 and 4.2 of this article.

More recently, Philip Warren Anderson (1923-2020) and Sir Nevill Francis Mott (1905-1996) developed new short-range theories to explain the superconducting state of a specific group of materials. Philip Warren Anderson and Ganapathy Baskaran, in 1987, proposed the Resonant Valence Bond (RVB) state mechanism to explain the superconducting state of cuprates La_2CuO_4 . In this mechanism, Cooper pairs of BCS Theory can be formed, but among others, the barrier of a possible instability of this resonant valence bond state was found in the formation of the superconducting ground state, which is defended in reference [[50],1989]. The explanation of the RVB Theory can be found in the reference [[51], 1987]. The other short-range theory was proposed by Sir Nevill Francis Mott in 1987 and became known as the Polaron Theory. This theory is an extrapolation of the BCS Theory to describe "High Temperature" superconductivity. The explanation of the Polaron Theory can be found in reference [[52], 1995].

Then, all these theories were elaborated for specific cases because they are phenomenological theories. Therefore, they are incomplete and short-range theories to try to describe the superconducting state of any material that may become a superconductor.

In general, through experimental arrangements, obtaining various results from many different materials, from the analysis of these results, a theory can be constructed as follows: **first step**, a mechanism is necessary to invent to trigger the interaction of particles or many bodies inside condensed matter, that generates the effects seen experimentally; **second step**, a Hamiltonian or Function must be elaborated to contain the information of the energies of the many particles; and **third step**, a Wave Function must be constructed to specify the state of movement, or energy, and so on, of the many particles. Thus, the second and third steps together can form one (or more) differential equations, which must be solved for the interior of the condensed matter under study.

Subsequently, the solutions obtained from these equations must agree with the results seen through experimental arrangements. The complete theory, which describes the superconducting state of condensed matter, can also be obtained by this route. Finally, the observant reader may be interested to know that this problem remains until unresolved the present moment.

6.3 Manufacture of the "High temperature" phase transition superconductor

Up to the present day, a superconducting material has not yet been manufactured experimentally, which exhibits its phase transition temperature within the same values of Earth's ambient temperature and pressure. This fact occurs because the exact mechanism or main experimental method that leads to

the emergence of superconductivity of particles within this temperature and pressure range has not yet been invented. However, superconductivity at “High temperatures” (473K=200oC), in hydrogen-rich compounds, which are called hydrides, has been projected its existence, but when submitted to enormous pressure. Nevertheless, this projection cannot be carried out on a large scale and utility because of this enormous pressure corresponds to the intensity of pressure at the core of planet Earth. Additionally, another barrier is that, recently, some articles published on this subject have been refuted by other scientific researchers in the area of superconductors. In this sense, at the end of Part 4 of this article, the observant reader will find a reference, which contains this barrier.

Then, this article is closed, making the observant reader conscious that the experimental construction of a superconducting state having a transition temperature equal to Earth's ambient temperature and pressure remains yet another unsolved problem for the scientific community to resolve. In this direction, Fig. 11 in Part 4 of this article shows the evolution of superconductor constructions from 1900 to 2015.

7. Acknowledgements and Statement of conflict of interest

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Additionally, the author declares that there are not potential conflicts of interest in the authorship and participation of the scientific work studied and prepared here for submission to institutions of foreign or national scientific publications. Grateful!

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