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Research Article

ANALYTICAL APPROACH TO CRUDE OIL WAX DEPOSITION PREDICTIONS IN PIPELINE

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Abstract

Wax deposition is one of the problems which are often encountered in oil pipeline transportation, depending on the Wax Appearance Temperature (WAT) of a particular flow stream. Although there has been some progress made over the years to understand this phenomenon, however, there are limited researches that can accurately account for all the factors that affect wax deposition especially in the wax simulators used presently in industries. This leaves a gap in research that needs to be explored thus making this research significant. This research was aimed at developing an analytical model for prediction of wax deposit taking into account both the thickness and positioning of the deposits within pipelines. A traditional equation of Pressure drop was modified and tested against an experimental model, and the Matzain model. Results from the new analytical model were compared to the results obtained from existing models for predicting wax deposition at flow temperatures of 15^0 C, 20^0 C and 30^0 C and they fell within accepted range. Hence, the new model can be used for predicting wax deposit points along pipelines.

Keywords: Wax deposition, Wax Appearance Temperature (WAT), Matzain model, Pipeline installation, Prediction equations.

INTRODUCTION

Paraffin wax deposition happens to be one of the age long threats to flow assurance in the transportation of produced crude in the sub-Saharan region. In Nigeria particularly, the problems are more serious offshore, where it is difficult to pressurize the system at intermediate points, leading to loss of flow lines and abandonment of wells (Adewusi, 1997). It is believed that most of the serious problems concerning wax deposition are the gelling and the start-up pressure requirements after shut downs, as well as the blocking of the entire open flow diameter (Ming, 2005). This causes the pipelines and production tubings to wax up beyond recovery necessitating frequent wax cutting which is quite expensive. Golczynski and Niesen (2001), stated that wax deposition is a steady state problem, in which paraffin is deposited slowly along the pipe walls when the wall temperature falls below the Wax Appearance Temperature (WAT) for an extended period of time. The WAT is the temperature at which wax crystals start appearing in the crude and is defined as the temperature at which Paraffin wax begins to solidify (McCain 1990). The ability to determine the severity of wax deposition is an extremely important issue, particularly in the design and development of deep-water oilfields. Although much progress has been made in the last decades to better the understanding of this complex process, the ability to accurately account for all the factors that affect wax deposition does not currently exist in the wax simulators used presently in industries. It is documented that the composition of crude oil influences the rate of wax depositions. Crude oil consists of various chemical components such as Paraffin, resins, bitumen, aromatics and naphthenes (Kjetil, 2011**).** The value of crude found in a reservoir is determined by how accessible the energy is, andcrude oil or hydrocarbons are usually sold to the end user as natural gas, gasoline, diesel or fuel oil. These products mainly consist of the smaller paraffin, ranging from C_1 up to C_{14}. The larger hydrocarbons need to be cracked in order to convert them into a saleable product, and this raises concerns on condensate generated from production site interfering with assured flow of oil towards processing facilities and refineries. This is because during the cracking of larger hydrocarbons, heavy molecules precipitate out of the oil or condensate due to reduction in pressure and temperature. These precipitates often deposit on production equipment such as flow lines and pipelines. This has necessitated the urgency to understand the fundamental variables that affect or influence wax deposition in order to improve optimum crude productivity.The main objective of this research was to develop an analytical model for prediction of wax deposition in pipelines, taking into consideration the thickness and point of deposition along the wall of the pipeline, as well as the Reynold number, friction factor of the fluid flow and changes in temperature along the pipeline.The model developed from this research, will assist in eliminating the requirement of too many data to predict wax deposition by older models, thus providing an alternative that can be easily assessable. The research was limited to the effect of temperature on certain properties of the crude oil such as density and viscosity, however the effect on the individual component of the hydrocarbon content of a flow stream and external factors such as possible pipeline leakage was not considered.

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METHODOLOGY

The methodology employed in this research was theoretical and analytical. The Pressure drop model was modified to obtain a new model for analytical prediction of wax deposit in pipelines. The new model developed was then subjected to simulated test with an experimental model and the Matzain model.

The Pressure Drop Method

The pressure drop method of calculating wax thickness was employed with the intention to compare deviation between experimental results and that obtained from theoretical examination. The pressure drop method predicts wax deposit thickness as functions of time for either every segment or the overall test section. The method is based on deposition reducing the hydraulic diameter of the pipe, which results in increased pressure drop for a constant flow rate operation. From momentum balance, the pressure gradient in a pipe segment can be obtained by the summation of the total pressure gradient as a result of friction, gravitational effect and acceleration effect, as shown in Equation 1;

$$\frac{dP}{dL} = \left(\frac{dP}{dL}\right)_f + \left(\frac{dP}{dL}\right)_g + \left(\frac{dP}{dL}\right)_{acc} = -\tau\frac{\pi d}{A} - \rho g sin\theta - \rho v\frac{dv}{dL} \quad (1)$$

However, for the purposes of flow in horizontal or near horizontal conditions, the pressure gradient due to gravitational effect can be ignored, therefore, leading to the following equation;

$$\frac{dP}{dL} = \left(\frac{dP}{dL}\right)_f = -\tau\frac{\pi d}{A} = -\frac{f\rho v^2}{2d_w} \quad (2)$$

$f = friction\, factor, \quad \rho = density\, of\, oil,$
$v = velocity\, and\, d_w = the\, diameter\, of\, flow$

If a reference time is chosen (say at the point where t equals to zero) and the current time is known, the thickness of the wax can be calculated from the following ratio between the pressure drops at the respective times;

$$\frac{\left(\frac{dP}{dL}\right)_t}{\left(\frac{dP}{dL}\right)_{ref}} = \frac{\frac{f_t\rho_t v_t^2}{2(d_w)_t}}{\frac{f_{ref}\rho_{ref}v_{ref}^2}{2(d_w)_{ref}}} \quad (3)$$

In the Equation 3, the reference time is assumed to have a zero thickness. The equation can be expressed as deposit thickness as in Equation 4;

$$\delta = \frac{d_i}{2}\left[1 - \left[\frac{f_t\rho_{ref}v_t^2}{f_{ref}\rho_t v_{ref}^2}\frac{\left(\frac{dP}{dL}\right)_{ref}}{\left(\frac{dP}{dL}\right)_t}\right]^{1/5}\right] \quad (4)$$

The Equation 4 can also be applied for laminar flow; the moody friction factor is given as;

$$for\ laminar\ flow;\ f = \frac{64}{N_{Re}} \quad (5)$$

$$\frac{1}{\sqrt{f}} = 1.74 - 2log_{10}\left(\frac{2\varepsilon}{d} + \frac{18.7}{N_{Re}\sqrt{f}}\right) \quad (6)$$

$\varepsilon = absolute\ pipe\ roughness, assumed\ to\ be\ 0.001\ for\ carbon\ stell\ pipe$

New Model – Modified Pressure Drop Method

$$\delta = \frac{d_i}{2}\left[1 - \left[\frac{f_t\rho_{ref}v_t^2}{f_{ref}\rho_t v_{ref}^2}\frac{\left(\frac{dP}{dL}\right)_{ref}}{\left(\frac{dP}{dL}\right)_t}\right]^{1/5}\right]$$

Rising to the fifth power, the both sides of Equation 7;

$$\delta^5 = \frac{d_i}{2}\left[1 - \left[\frac{f_t\rho_{ref}v_t^2}{f_{ref}\rho_t v_{ref}^2}\frac{\left(\frac{dP}{dL}\right)_{ref}}{\left(\frac{dP}{dL}\right)_t}\right]^{1/5}\right]^5$$

$$\delta^5 = \left(\frac{d_i}{2}\right)^5\left(1 - \frac{f_t\rho_{ref}v_t^2}{f_{ref}\rho_t v_{ref}^2}\frac{\left(\frac{dP}{dL}\right)_{ref}}{\left(\frac{dP}{dL}\right)_t}\right) \quad (8)$$

$$\delta^5 = \left(\frac{d_i}{2}\right)^5 - \left(\frac{d_i}{2}\right)^5\frac{f_t\rho_{ref}v_t^2}{f_{ref}\rho_t v_{ref}^2}\frac{\left(\frac{dP}{dL}\right)_{ref}}{\left(\frac{dP}{dL}\right)_t} \quad (9)$$

$$\frac{\left(\frac{d_i}{2}\right)^5 - \delta^5}{\left(\left(\frac{d_i}{2}\right)^5\frac{f_t\rho_{ref}v_t^2}{f_{ref}\rho_t v_{ref}^2}\right)} = \frac{\left(\frac{dP}{dL}\right)_{ref}}{\left(\frac{dP}{dL}\right)_t} \quad (10)$$

Taking the right side of the Equation 10

$$\frac{\left(\frac{dP}{dL}\right)_{ref}}{\left(\frac{dP}{dL}\right)_t} \quad (11)$$

$\frac{dP}{dL} is the pressure drop over the distance dL moved by a specified volume of fluid$

$$\frac{dP}{dL} = \Delta P \quad (12)$$

$$\left(\frac{dP}{dL}\right)_{ref} = (\Delta P)_{ref} \quad (13)$$

$$\left(\frac{dP}{dL}\right)_t = (\Delta P)_t \quad (14)$$

Therefore, the right hand side of Equation 10 can be re-written as;

$$\frac{\left(\frac{dP}{dL}\right)_{ref}}{\left(\frac{dP}{dL}\right)_t} = \frac{(\Delta P)_{ref}}{(\Delta P)_t} \quad (15)$$

From Hooks law of elasticity;

Stress acting on a material is directly proportional to strain, with young's modulus of the material as the constant of proportionality.

$$\sigma = E\,\varepsilon \quad (16)$$

$$\sigma = stress = \frac{Force}{Area} \quad (17)$$

$E = Young's moduls\ of\ eleasttcity$

$$\varepsilon = strain = \frac{\Delta L}{L} = \frac{change\ in\ lenght}{origianal\ leght} \quad (18)$$

Similarly, from the knowledge of cubic expansivity of materials

Cubic expansivity is a material's tendency to increase in volume in response to an increase in temperature. Cubic expansivity is a type of thermal expansion. It is described by a fraction that represents the fractional increase in volume of fluid exposed to a temperature increase of one degree Celsius. The fraction is called the cubic expansion coefficient of the material

$$\Delta V = \beta V_1 \Delta T \quad (19)$$

Combining Equations 16 and 18

$$\sigma = E \times \frac{\Delta L}{L} \quad (20)$$

In the case of liquid and cubic expansivity, stress can be represented as in Equation 21;

$$\sigma = E \times \frac{\Delta V}{V} \quad (21)$$

Substituting Equation 19 in Equation 21

$$\sigma = E \times \frac{\beta V_1 \Delta T}{V} \quad (22)$$

Take note from Equation 22 that $V_1 = V = initial\ volume$ Hence, Equation 22 becomes;

$$\sigma = E \times \beta \Delta T \quad (23)$$

Making ΔT the subject of the formula gives;

$$\Delta T = \frac{\sigma}{\beta E} = \frac{F}{A\beta E} \quad (24)$$

From the Equation 24, change in temperature is directly proportional to induced stress due to pressure change. It therefore implies that

$$\Delta T \propto (\sigma = \frac{F}{A} = \Delta P) \quad (25)$$

$$\Delta T \propto \Delta P \quad (26)$$

$$it\ therefore\ means\ that;\ \frac{\Delta P_1}{\Delta T_1} = \frac{\Delta P_2}{\Delta T_2} \quad (27)$$

$$Hence;\ \frac{\Delta P_{ref}}{\Delta T_{ref}} = \frac{\Delta P_t}{\Delta T_t} \quad (28)$$

$$rearranging;\ \frac{\Delta P_{ref}}{\Delta P_t} = \frac{\Delta T_{ref}}{\Delta T_t} \quad (29)$$

Combining Equation (10), (15), and (29)

$$\frac{\left(\frac{d_i}{2}\right)^5 - \delta^5}{\left(\left(\frac{d_i}{2}\right)^5 \frac{f_t \rho_{ref} v_t^2}{f_{ref} \rho_t v_{ref}^2}\right)} = \frac{\Delta T_{ref}}{\Delta T_t} \quad (30)$$

Re-arranging to make wax thickness the subject of the formula;

$$\delta = \frac{d_i}{2}\left[1 - \left[\frac{f_t \rho_{ref} v_t^2}{f_{ref} \rho_t v_{ref}^2} \frac{\Delta T_{ref}}{\Delta T_t}\right]^{\frac{1}{5}}\right]$$

Applying the Modified Pressure Drop Model

The analytical model is intended to determine the point along the pipeline where wax will begin to deposit and at the same time measure the thickness of such deposit.

From figure 2 shownabove, the segmented pipeline has 5 segments. Also, the number of segments is determined by the user of the model after which the temperature drop in each segment of the pipelineis calculated out.

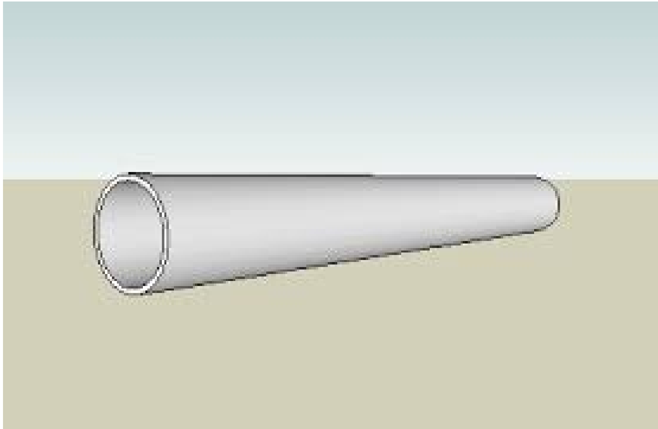

Figure 1. Typical horizontal Pipe

Step 1: The heat transfer model:

Depending on the type of pipe, the temperature in each segment can be calculated using the Composite, non-composite or insulated pipe method as follows;

For a Composite pipe

$$Q = (T_1 - T_2)2\pi L * \frac{1}{\left[\frac{1}{k_1}\text{in}\left(\frac{r_1}{r_i}\right) + \frac{1}{k_2}\text{in}\left(\frac{r_2}{r_1}\right) + \frac{1}{k_3}\text{in}\left(\frac{r_o}{r_2}\right)\right]} \quad (32)$$

$$T_1 = T_2 + \frac{Q}{2\pi l}\left[\frac{1}{k_1}\text{in}\left(\frac{r_1}{r_i}\right) + \frac{1}{k_2}\text{in}\left(\frac{r_2}{r_1}\right) + \frac{1}{k_3}\text{in}\left(\frac{r_o}{r_2}\right)\right] \quad (33)$$

For the composite section, the user should add the number of materials making up the composite, before it can decide whether to calculate for two or more materials as shown below;

For an insulated pipe

$$Q = (T_1 - T_2)/[(\ln(r_o/r_i)/2\pi kL) + (\ln(r_s/r_o)/2\pi k_s L)] \quad (34)$$

$$T_1 = {}^{Q}/_{[(\ln(r_o/r_i)/2\pi kL) + (\ln(r_s/r_o)/2\pi k_s L)]} + T_2 \quad (35)$$

For a non-composite pipe

$$Q = 2\pi kl \times \frac{(T_1 - T_2)}{\left[\ln\frac{r_2}{r_1}\right]} \quad (36)$$

$$T_1 = T_2 + \frac{Q \times \left[\ln\frac{r_2}{r_1}\right]}{2\pi kl} \quad (37)$$

Where: Equation (31)

T_1 = inlet Temperature in K,
T_2 = Outside Temperature of the pipe,
Q = Heat transfer rate in J/s or Watt,
$k_{1,2,3}$ = thermal conductivity of respective material through which the heat flows in $Wm^{-1}K^{-1}$,
r_2 =outer radius of the pipe,
r_1 =inner radius of the pipe.
L= length of the pipe.
$\pi = 3.14159$

It is important to note that this step will be repeated for all the points except point 1 (i.e 2, 3, 4….) depending on the number of points. From the example in Figure 2, it will be for points, 2, 3, 4, 5 and 6.For each of the points, T_2 is the outside temperature of the pipe at that point. (It should also be noted that for each of the point, L, will be measured from the beginning of the pipe e.g for point 3, L, will be length of pipe measured from point 0 to point 3). The output temperature at each point along with the length is presented in Table 1 below;

Table 1. Sample Pattern for tabulating result obtained from the heat transfer Model

segment no./point	Length of pipe from Origin to point of interest	Temperature value at point of interest
Segment 1- Point 1	Length of Point 0- Point 1	Avg Temp in Segment 1
Segment 2- Point 2	Length of Point 0- Point 2	Avg Temp in Segment 2
Segment 3- Point 3	Length of Point 0- Point 3	Avg Temp in Segment 3
Segment 4- Point 4	Length of Point 0- Point 4	Avg Temp in Segment 4
Segment 5-Point 5	Length of Point 0- Point 5	Avg Temp in Segment 5
Segment 6- Point 6	Length of Point 0- Point 6	Avg Temp in Segment 6

Once the temperature values are available, then further calculations can continue. For instance,

When applying the Modified Wax deposition Pressure Drop Model, the following parameters represented in the equation has to be calculated;

i. density
ii. velocity and
iii. friction factor

$$\delta = \frac{d_i}{2}\left[1 - \left[\frac{f_t \rho_{ref} v_t^2}{f_{ref} \rho_t v_{ref}^2}\frac{\Delta T_{ref}}{\Delta T_t}\right]^{1/5}\right] \quad (31)$$

$d_i = internaldiameterofpipe\ (measured)$

Step 2: Determination of velocity (v_t and v_{ref})

v_t = velocity of flow at temperature/point of interest

$v = \frac{Q}{A}$

Q = flow rate of crude oil

A= area of pipeline (cylinder) up to the point of interest

$A = 2\pi rh$

$v = \frac{Q}{2\pi rh} = \frac{Q}{2\pi rL}$

NB: Note that h in the Equations 38, 39 and 40 is L for each of the points

Since every temperature has a corresponding length measured, then velocity at that temperature will be gotten by using the length measured at the point where temperature was observed

Step 3: Density determination (ρ_t and ρ_{ref})

For this step, the chart and equations following will be used by the user of the model

The API value of the crude will be used in Equation 2.41;

$\rho_{60} = \left(\frac{141.5}{131.5 + API}\right) 62.37$

The "DCF" can be obtained from the chart above and used to determine the density of the crude oil at the point of interest as follows

$\rho_t = DCF \times \rho_{60}$

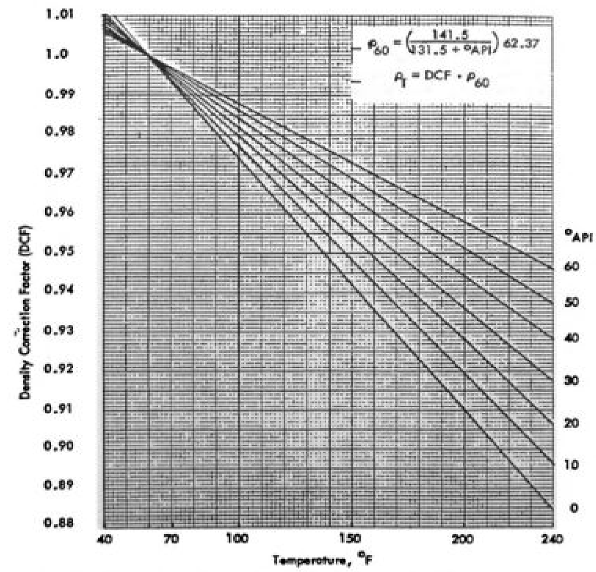

Figure 3. Effect of Temperature on Density of Crude oil (Hankinson and Thompson 1979)

If the temperature of the crude oil is less than 40^{0}F, the chart can no longer be used then Equation 43 should be used in determining the density;

Get API gravity of the crude oil

Use the equation;

$S.G = Specific\ gravity = \frac{141.5}{(131.5 + API)}$

Calculate the initial density of crude oil before the new temperature using;

$\rho_{ref} = S.G * \rho_w$

Then use the initial density or reference density (ρ_{ref}) to determine the density at the temperaute under consideration; Equation (38)

$\rho_T = \frac{\rho_{ref}}{[\{(T_1 - T_2)0.001\} + 1]}$ Equation (39)

Equation (40)

Step 4: Friction Factor determination f_t&f_{ref})

For turbulent flows;

$$f_t = \left\{\frac{1}{-4log\left(\frac{\varepsilon}{3.7065} - \frac{5.0452}{N_{RE}}\log\left\{\frac{\varepsilon^{1.1098}}{2.8257} + \left(\frac{7.149}{N_{RE}}\right)^{0.8981}\right\}\right)}\right\}^2$$

$\varepsilon = pipe\ roughness$

For Laminar Flows;

$f = \frac{64}{N_{Re}}$

General (i.e for both Laminar and Turbulent)

Equation (41)

$N_{RE} = \frac{Dv\rho}{\mu}$

$\mu = \left(\frac{3.141 \times 10^{10}}{T^{3.444}}\right)\log(\gamma_{API})^{[10.313\ logT - 36.4447]}$

$D = pipe\ internal\ diameter$

$v = velocity\ at\ the\ point\ under\ consideration\ as\ obtained\ above$

$\rho = density\ of\ crude\ oil\ at\ the\ point\ as\ obtained\ from\ above$

$\mu = viscosity$ Equation (42)

Step 5: Temperature determination T_t& T_{ref}

$\Delta T_t = temperatureatthepointunderconsideration$
$\Delta T_{ref} = temperatureatthepreviouspoint$
Note: "ref" simply refers to parameters calculated at previous points.

Step 6: Determination of Duration of flow

The duration of flow at each point can be calculated as follows;

$$Time\ (duration\ of\ flow) = \frac{Distance\ @\ point\ of\ consideration}{veocity\ of\ flow\ at\ point\ of\ consideration}$$

Equation (50)

Substituting all the parameters into the equation;

$$\delta = \frac{d_i}{2}\left[1 - \left[\frac{f_t \rho_{ref} v_t^2}{f_{ref} \rho_t v_{ref}^2} \frac{\Delta T_{ref}}{\Delta T_t}\right]^{1/5}\right]$$

Equation (51)

The following results among others can be obtained by plotting wax thickness against duration to see trend orPlotting Wax thickness against length of pipe to see the point where wax deposit will impact on flow assurance.

The comparative analysis

In order to carry out the comparative analysis, we compared results from the Matzain model for wax deposition, pressure drop model and results from modified pressure drop equation.

Experimental Data

An experiment previously performed in the wax rig in the work carried out by Karianne (2008) wasused for further comparison. Table 2 shows the condensate temperature, water temperature, and condensate flow rate and time duration for the various experiments used for comparison. The temperature of the condensate was 15^{O}C, 20 ^{O}C and 30 ^{O}C. All of these temperatures were below the WAT, which is 45°C. The pipe was cooled by water at 10°C, and thus the temperature difference between condensate and water was in the range of 5°C to 20 °C. The condensate flow rate was 21m^3/h.

Table 2. Wax Rig Data

		UNIT		UNIT
Pi	3.14			
OIL PIPE ID	52.58	mm	0.05258	m
OIL PIPE IR	26.29	mm	0.02629	m
oil pipe OD	60.56	mm	0.06056	m
Oil pipe OR	30.28	mm	0.03028	m
Water Pipe, ID	131.33	mm		
Fluid	Waxy condensate			
WAT	45	^{o}c		
Length of Pipe	5000	mm	5	m
heat transfer coefficient for steel	43	w/mk		
API	43	^{O}API		
SG		SG	0.8108883	
		Initial density	0.8108883	kg/m^3

Table 3. Experimental Test Matrix

Experiment	Condensate Temperature	Water Temperature	Condensate flow rate m^3/h
15_10_21	15	10	21
20_10_21	20	10	21
30_10_21	30	10	21

Ethical Issues

In the course of carrying out this research, the authors ensured that no ethical issues relating to humans, animal or the environment was violated, since the research was entirely theoretical and not experimental. The data obtained were not doctored and the benefits of the research were not exaggerated.

RESULTS

The modified Pressure drop model was tested using the conditions presented in the experimental test matrix (Table 3) and the data extractedfrom wax rig data (Table 2). The segmented pipe presented in Figure 1 was also tested and the results obtained are tabulated as follows.

Table 4. Results for modified pressure drop method at a flow temperature of 15^{0}c

T_t	Velocity	Density (T_t)	Viscosity	ID	REY NO.	Fric-Factor	Wax-dep	Distance	Time
15	127.1947	0.8108	11.9260	0.0525	15741.06	0.0073	0	1	0.0079
12.5	63.5970	0.8088	14.7526	0.0525	12693.39	0.0076	0.0054	2	0.0315
11.667	42.3980	0.8081	16.0077	0.0525	11688.35	0.0078	0.0082	3	0.0708
11.25	31.7990	0.8078	16.6820	0.0525	11211.26	0.00789	0.0100	4	0.1258
11	25.4390	0.8076	17.1251	0.0525	10918.43	0.0079	0.0113	5	0.1966

Table 5. Results for modified pressure drop method at a flow temperature of 20^{0}c

T_t	velocity	density (T_t)	viscosity	ID	REY NO.	Fric-Factor	Wax-dep	Distance	Time
20	127.1947	0.8108	8.5259	0.0525	44008.82	0.0061	0	1	0.0079
15	63.5970	0.8068	11.9260	0.0525	31305.55	0.0064	0.0049	2	0.0315
13.33	42.3980	0.8055	13.6826	0.0525	27241.06	0.0066	0.0076	3	0.0708
12.5	31.7990	0.8048	14.7526	0.0525	25244.35	0.0067	0.0093	4	0.1258
12	25.4390	0.8044	15.4721	0.0525	24058.3	0.00676	0.0106	5	0.1966

Table 6: Results for modified pressure drop method at a flow temperature of 30^{0}c

T_t	velocity	density (T_t)	viscosity	ID	REY NO.	Fric-Factor	Wax-dep	Distance	Time
30	127.1947	0.8108	5.3127	0.0525	141252.5	0.0053	0	1	0.0079
20	63.5970	0.8028	8.5259	0.0525	87146.2	0.0056	0.0044	2	0.0315
16.667	42.3980	0.8001	10.5466	0.0525	70215.2	0.0057	0.0069	3	0.0708
15	31.7990	0.7988	11.9260	0.0525	61990.86	0.0058	0.0086	4	0.1258
	25.4390	0.7980	12.9256	0.0525	57139.81	0.0059	0.0098	5	0.1966

Table 7. Wax Deposit Thickness from Experimental Case Studies at varying Temperatures of 15^0C, 20^0C and 30^0C (Karianne, 2008)

Velocity	Distance	Time	15^0C	20^0C	30^0C
127.1947	1	0.0079	0	0	0
63.5970	2	0.0315	0.0056	0.005	0.0048
42.3980	3	0.0708	0.0100	0.008	0.007
31.7990	4	0.1258	0.0120	0.009	0.008
25.4390	5	0.1966	0.0130	0.011	0.009

Table 8. Wax Deposit thickness from Matzain simulated at varying temperatures of 15^0C, 20^0C and 30^0C

Velocity	Distance	Time	15^0C	20^0C	30^0C
127.1947	1	0.0079	0	0	0
63.5970	2	0.0315	0.051	0.0042	0.004
42.3980	3	0.0708	0.078	0.0072	0.0058
31.7990	4	0.1258	0.08	0.0088	0.0068
25.4390	5	0.1966	0.088	0.009	0.0075

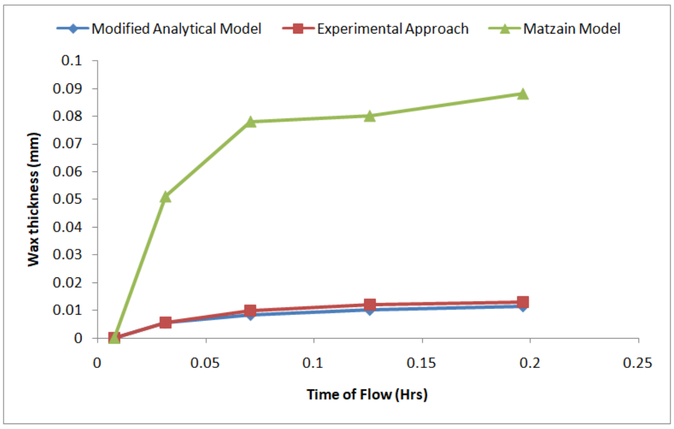

Figure 4. Combined flow for all three models @ 15^0C

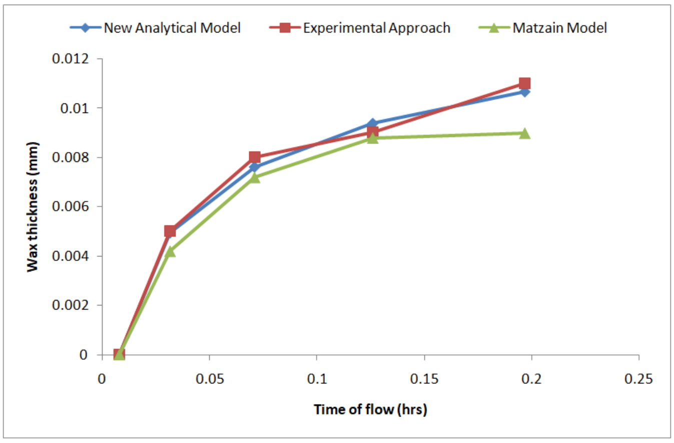

Figure 5. Combined flow for all three models @ 20^0C

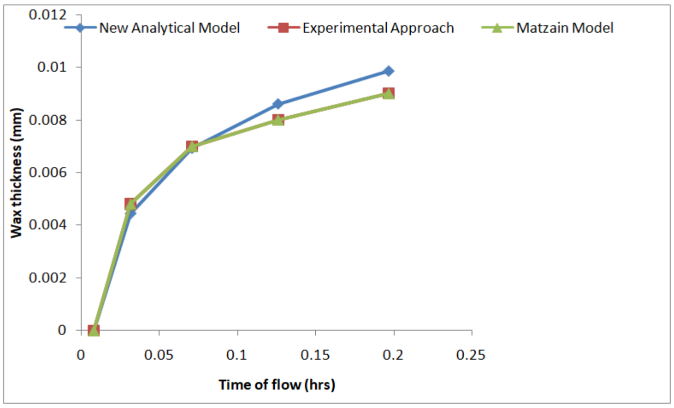

Figure 6. Combined flow for all three models @ 30^0C

DISCUSSION AND CONCLUSION

Analysis of result obtained from all three models at 15^0C

Findings from the study revealed that based on the wax deposition prediction made using the modified pressure drop equation, the experimental method, and Matzain simulation at temperature of 15^0C (Tables4, 7 and 8), it was evident that the modified pressure drop model and Experimental method predicted approximately the same values. However, the Matzain model showed a slight difference as it predicts slightly higher values compared to the other two models as shown in Figure4.This is due to the assumptions made in the Matzain model as documented in the work conducted by Matzain et al (2001). Also, the shape of the curve obtained with the modified pressure drop model is similar to that of the Matzain model; this is because the trends of wax deposition seem to be the same thing in all cases.

Analysis of result obtained from all three models at 20^0C

Findings from the study revealed that based on the wax deposition prediction made using the modified pressure drop equation, the experimental method, and the Matzain simulation at temperature of 20^0C (Tables5, 7 and 8), all three models predicted lower deposition at the beginning of the flow when compared to the results obtained during the input temperature of 15^0C (Figure5). The reason for this deviation observed at 20^0C when compared to predictions at 15^0C is primarily due to increase in temperature of the flow stream at each of the point. However the pressure drop equation and the experimental result predicted same, this further shows the closeness of the modified method to real life scenario.Going by this, previous discrepancies could be attributed to human error during experimental process and may also be due to factors not considered in the models which could have been taken care of in the experimental process. The Matzain simulation still predicted lower values compared to the rest of the results.

Analysis of result obtained from all three models at 30^0C

In order to understand the discrepancies, even though minute, and within the acceptable range for scientific purposes, the process was again repeated using input temperatures of 30^0C. And the Findings revealed that at temperature of 30^0C (Tables6, 7 and 8), the temperature difference becomes higher, and the wax deposit is reduced (Figures 6). This is because wax deposition is dependent on WAT and this observation supports the work done by Hamouda and Viken(1993).It was further observed that the modified method of engaging analytical method to determine wax deposition is approximately equal to values obtained in the experimental processes. And when compared to values obtained from the Matzain model, the values are slightly different but within acceptable range Figure 6. Therefore, for this models, as temperature is increased, the large discrepancy in temperature between the surrounding and environment reduces the rate at which the flow stream reaches the wax appearance temperature (WAT) and hence the reason for the low wax deposition prediction. The authors further believe that the slight difference in values compared to that of experimental values could be possibly due to situations such as leakages during high temperature run. However, it is important to note that considering the degree of disparity of prediction between all three models from 15^0C to 30^0C, it is evident that the modified pressure drop equation is effective for the purpose of predicting wax deposition as well as points along the flow line where wax begin to deposit, and the point where it will become a major challenge if not tackled early enough. In conclusion, the research aimed at developing an analytical model that can be used to effectively analyze crude oil wax deposition predictions in pipeline. The pressure drop method and the Matzain model were selected due to their less cumbersome nature and the nearness to real life situation. The pressure drop method was modified to make it adaptable for predicting wax deposit without the need of any laboratory experiment. The results from the new model (modified pressure drop method) was further compared to that obtained from the Matzain method and the experimental method. The results showed that with the newly developed model, values obtained were approximately equal to those from experimental studies. Also the result obtained was within acceptable range when compared to the Matzain method of predicting wax deposition. This suggests that the newly developed method can be used to predict point of wax appearance in a pipeline as well as the thickness of the wax deposited at the point under consideration. The outcome of this study will help to determine injection points for inhibitors and also in planning treatment for enhancing waxy oil production.

Recommendation

It is important that further research be carried out in order to investigate the possible behavior of the newly developed model when flow stream is exposed to certain conditions such as leakages along the pipeline. Also, further studies need to be carried out in order to investigate the use of possible temperature correction factor or how to minimize errors which could be due to approximation of temperature values when using the new model.

Data Availability Statement: All data, models, and code generated or used during the study appear in the published article.

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