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Research Article

OPTIMAL SOLUTION OF SYMMETRIC TRIANGULAR FUZZY TRANSPORTATION PROBLEM USING STEPPING STONE METHOD

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Abstract

To main objective of this paper is to find the optimal solution for the fuzzy transportation problem using stepping stone method. In the proposed method, transportation cost, availability and demand of the product are represented by symmetric triangular fuzzy numbers (STFNs). The STFNs are defuzzificated by proposed ranking technique. The initial basic feasible solution is obtained by vogel's approximation method (VAM) and then the optimal solution is obtained by stepping stone method. A numerical example is given to show the optimal solution.

Keywords: Fuzzy number, Triangular fuzzy number, Symmetric Triangular fuzzy number, Ranking technique, Transportation problem.

INTRODUCTION

Transportation problem [TP] is significant type of the linear programming problem which is related with day-to-day real life situations. In transportation problem may be uncertain due to some uncontrollable factor. To overcome this, fuzzy set was introduced by Zadeh in 1965. After this prelusion work several authors such as Shiang-Tai Liu and Chiang Kao *et al*. (2006), Chanas *et al*. (1989), Pandian *et al*. 2010, Liu and Kao 2004 etc proposed different methods for the solution of Fuzzy transportation problems. Amarpreet Kaur and Amit Kumar, (2011) presented a new algorithm for solving fuzzy transportation problem using ranking function. Later, different ranking technique was used to solve TP (Iden Hasan Hussein and Anfal Hasan Dheyab, 2015; Poonam Shugani and Abbas Vijay Gupta, 2012; Shugani Poonam *et al.,* 2015; Surjeet Singh Chauhan and Nidhi Joshi, 2013). Nareshkumar and Kumara Ghuru were discussed a initial basic feasible solution of fuzzy transportation problem in 2014. In this paper, we discuss the optimal solution of fuzzy transportation problem involving symmetric triangular fuzzy number(STFN) using stepping stone method. The paper organized as follows, First in section2, we recall the basic terminologies and some operations of STFNs. In section3, we discuss the mathematical formulation of fuzzy transportation problem. In section4, we define the procedure for solving transportation problem using proposed ranking function. In section5, we have been presented the example of proposed method. In section6, conclusion is given.

Preliminaries

Triangular fuzzy number (Jaisankar *et al.,* 2018): A fuzzy number $\grave{A}$ is defined to be a triangular fuzzy numbers if its membership function $\alpha_{\grave{A}}(x) = X \rightarrow [0,1]$ is defined by,

$$\alpha_{\grave{A}}(x) = \begin{cases} 0 & if\ x \le p_1 \\ \dfrac{x-p_1}{p_2-p_1} & if\ p_1 \le x \le p_2 \\ 1 & if\ \ x = p_2 \\ \dfrac{p_3-x}{p_3-p_2} & if\ p_2 \le x \le p_3 \\ 0 & if\ x > p_3 \end{cases}$$

It is denoted by $\grave{A} = (p_1, p_2, p_3)$

Symmetric Triangular Fuzzy Number (STFN) (Nareshkumar *et al.,* 2014): If $p_2 = p_3$, then the triangular fuzzy number $\grave{A} = (p_1, p_2, p_3)$ is called symmetric triangular fuzzy number. It is represented by $\grave{A} = (p_1, p_2)$, where p_1 is core $\grave{A}$, p_2 is left breadth and right breadth of c.

Ranking of Symmetric Triangular Fuzzy Number (Nareshkumar *et al.,* 2014): If $\grave{A} = (p_1, p_2, p_3)$ is the symmetric triangular fuzzy number then ranking of STFN is represented as

$$R(\grave{A}) = \frac{1}{6}(p_1 + 4p_2 + p_3)$$

Arithmetic Operations (Jaisankar *et al.,* 2018)

- If $\grave{A} = (p_1, p_2, p_3)$ and $\grave{B} = (g_1, g_2, g_3)$ then
$$\grave{A} + \grave{B} = (p_1 + g_1, p_2 + g_2, p_3 + g_3)$$
- If $\grave{A} = (p_1, p_2, p_3)$ and $\grave{B} = (g_1, g_2, g_3)$ then
$$\grave{A} - \grave{B} = (p_1 - g_3, p_2 - g_2, p_3 - g_1)$$
- If $\grave{A} = (p_1, p_2, p_3)$ and $\grave{B} = (g_1, g_2, g_3)$ then
$$\grave{A}\grave{B} = (min[p_1g_1, p_1g_3, p_1g_1, p_3g_3], p_2g_2, max[p_1g_1, pg_3, p_3g_1, p_3g_3]).$$

Fuzzy transportation problem (Jaisankar *et al.,* 2018):

Let a_i be the quantity of commodity available at origin i. b_j be the quantity of commodity needed at destination j. Let $\grave{C}_{ij}$ be the fuzzy cost of transporting one unit of commodity from origin i to destination j. Then the problem is to determine the transportation schedule so as to minimize the total fuzzy transportation cost satisfy supply and demand constraints. The mathematical model of fuzzy transportation problem is given by

Minimize $\hat{\boldsymbol{z}} = \sum_{i=1}^{n} \sum_{j=1}^{n} \grave{\boldsymbol{C}}_{ij} \grave{\boldsymbol{x}}_{ij}$

Subject to
$\sum_{j=1}^{n} \grave{\boldsymbol{x}}_{ij} = \boldsymbol{a}_i$, $\boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}, \ldots \boldsymbol{n};$

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$\sum_{i=1}^{n} \grave{x}_{ij} = b_j \;, j = 1, 2, \ldots n$
$\grave{x}_{ij} \geq 0$ for all i **and** j
$\grave{C}_{ij} = \grave{C}_{ij}^{1}, \grave{C}_{ij}^{2}, \grave{C}_{ij}^{2}$

In the above model, the transportation cost $\grave{C}_{ij}$, Supplies a_i, and the demands b_j are symmetric triangular fuzzy number.

Procedure for solving fuzzy transportation problem

We will present a solution to fuzzy shipping problem including moving cost, supply and demand costs are referred as symmetric triangular fuzzy numbers.

Step 1: First change the cost, requirement and provide principles which are all Symmetric Triangular Fuzzy numbers into crisp values by using proposed ranking technique.

Step 2: Examine the condition that the converted fuzzy transportation problem is balanced or not

- (i). If is balanced go to Step 4.
(Total supply = Total demand).
- (ii). If not balanced go to step 3.
(Total supply ≠ Total demand).

Step 3: If the convert Fuzzy transportation problem is not balanced then add dummy row (or) dummy column with cost value as zero and supply value or demand value as(according to row or column) the difference between the total supply and total demand value

Step 4: Obtain the initial basic feasible solution [IBFS] using VAM method.

Step 5: Find the optimal solution by stepping stone method.

Numerical example

Consider a fuzzy transportation problem whose cost, supply and demand values are symmetrical triangular fuzzy number.

	D1	D2	D3	D4	Demand
S1	(4,8,8)	(0,5,5)	(0,5,5)	(0,5,5)	(2,8,8)
S2	(1,4,4)	(2,3,3)	(2,3,3)	(2,3,3)	(0,3,3)
S3	(1,4,4)	(2,5,5)	(4,8,8)	(4,9,9)	(2,13,13)
Supply	(2,4,4)	(1,4,4)	(0,9,9)	(1,7,7)	

Total demand = Total supply = (4,24,24)

The given problem is balanced
Find the ranking value

$a_{11} = 7.33$; $a_{12} = 4.16$;$a_{13} = 4.16$;$a_{14} = 4.16$; $d_1 = 7$;
$a_{21} = 3.5$; $a_{22} = 2.83$;$a_{23} = 2.83$;$a_{24} = 2.83$; $d_2 = 2.5$
$a_{31} = 3.5$; $a_{32} = 4.5$;$a_{33} = 7.33$;$a_{34} = 8.16$; $d_3 = 11.16$
$s_1 = 3.66$; $s_2 = 3.5$; $s_3 = 7.5$; $s_4 = 6$

	D1	D2	D3	D4	Demand
S1	7.33	4.16	4.16	4.16	7
S2	3.5	2.83	2.83	2.83	2.5
S3	3.5	4.5	7.33	8.16	11.16
Supply	3.66	3.5	7.5	6	

Using the VAM method, The initial feasible solution is as follows

<table>
<tr><th></th><th>D1</th><th>D2</th><th>D3</th><th>D4</th><th>Supply</th><th>Row Penalty</th></tr>
<tr><td>S1</td><td>7.33</td><td>4.16</td><td>4.16(1)</td><td>4.16(6)</td><td>7</td><td>0 | 0 | 0 | 0 | -- | -- |</td></tr>
<tr><td>S2</td><td>3.5</td><td>2.83(2.5)</td><td>2.83</td><td>2.83</td><td>2.5</td><td>0 | -- | -- | -- | -- | -- |</td></tr>
<tr><td>S3</td><td>3.5(3.66)</td><td>4.5(1)</td><td>7.33(6.5)</td><td>8.16</td><td>11.16</td><td>1 | 1 | 1 | 2.83 | 2.83 | 4.5 |</td></tr>
<tr><td>Demand</td><td>3.66</td><td>3.5</td><td>7.5</td><td>6</td><td></td><td></td></tr>
<tr><td>Column Penalty</td><td>0
3.8
3.8
--
--
--</td><td>1.33
0.34
0.34
0.34
4.5
4.5</td><td>1.33
3.17
3.17
3.17
7.33
--</td><td>1.33
4
--
--
--
--</td><td></td><td></td></tr>
</table>

The initial basic feasible solution is

=4.16×1+4.16×6+2.83×2.5+3.5×3.66+4.5×1+7.33×6.5=**101.15**

Here, the number of allocated cells = 6.

It is equal to m + n - 1 = 3 + 4 - 1 = 6

∴ This solution is non-degenerate.

Now we find the optimum solution. The allocation Table of the IBFS is,

	D1	D2	D3	D4	Supply
S1	7.33	4.16	4.16 (1)	4.16 (6)	7
S2	3.5	2.83 (2.5)	2.83	2.83	2.5
S3	3.5 (3.66)	4.5 (1)	7.33 (6.5)	8.16	11.16
Demand	3.66	3.5	7.5	6	

Iteration 1

1. Create closed loop for unoccupied cells, we get

Unoccupied cell	Closed path	Net cost change
S1D1	S1D1→S1D3→S3D3→S3D1	7.33 - 4.16 + 7.33 - 3.5=7
S1D2	S1D2→S1D3→S3D3→S3D2	4.16 - 4.16 + 7.33 - 4.5=2.83
S2D1	S2D1→S2D2→S3D2→S3D1	3.5 - 2.83 + 4.5 - 3.5=1.67
S2D3	S2D3→S2D2→S3D2→S3D3	2.83 - 2.83 + 4.5 - 7.33=-2.83
S2D4	S2D4→S2D2→S3D2→S3D3→ S1D3→S1D4	2.83 - 2.83 + 4.5 - 7.33 + 4.16 - 4.16=-2.83
S3D4	S3D4→S3D3→S1D3→S1D4	8.16 - 7.33 + 4.16 - 4.16=0.83

2. Select the unoccupied cell having the highest negative net cost change i.e. cell S2D3=-2.83.and draw a closed path from S2D3.

Closed path is S2D3→S2D2→S3D2→S3D3

Closed path and plus/minus allocation for current unoccupied cell S2D3

	D1	D2	D3	D4	Supply
S1	7.33 [6.97]	4.16 [2.83]	4.16 (1)	4.16 (6)	7
S2	3.5 [1.67]	2.83 (2.5) (-)	2.83 [-2.83] (+)	2.83 [-2.83]	2.5
S3	3.5 (3.66)	4.5 (1) (+)	7.33 (6.5) (-)	8.16 [0.83]	11.16
Demand	3.66	3.5	7.5	6	

3. Minimum allocated value among all negative position (-) on closed path = 2.5. Subtract 2.5 from all (-) and Add it to all (+)

	D1	D2	D3	D4	Supply
S1	7.33	4.16	4.16 (1)	4.16 (6)	7
S2	3.5	2.83	2.83 (2.5)	2.83	2.5
S3	3.5 (3.66)	4.5 (3.5)	7.33 (4)	8.16	11.16
Demand	3.66	3.5	7.5	6	

4. Repeat the above steps, until an optimal solution is obtained.

Iteration 2

1. Create closed loop for unoccupied cells, we get

Unoccupied cell	Closed path	Net cost change
S1D1	S1D1→S1D3→S3D3→S3D1	7.33 - 4.16 + 7.33 - 3.5=7
S1D2	S1D2→S1D3→S3D3→S3D2	4.16 - 4.16 + 7.33 - 4.5=2.83
S2D1	S2D1→S2D3→S3D3→S3D1	3.5 - 2.83 + 7.33 - 3.5=4.5
S2D2	S2D2→S2D3→S3D3→S3D2	2.83 - 2.83 + 7.33 - 4.5=2.83
S2D4	S2D4→S2D3→S1D3→S1D4	2.83 - 2.83 + 4.16 - 4.16=0
S3D4	S3D4→S3D3→S1D3→S1D4	8.16 - 7.33 + 4.16 - 4.16=0.83

Since all net cost change ≥0
so final optimal solution is arrived.

	D1	D2	D3	D4	Supply
S1	7.33	4.16	4.16 (1)	4.16 (6)	7
S2	3.5	2.83	2.83 (2.5)	2.83	2.5
S3	3.5 (3.66)	4.5 (3.5)	7.33 (4)	8.16	11.16
Demand	3.66	3.5	7.5	6	

The optimal total transportation cost
$=4.16\times1+4.16\times6+2.83\times2.5+3.5\times3.66+4.5\times3.5+7.33\times4=\mathbf{94.07}$

Notice alternate solution is available with unoccupied cell S2D4=0, but with the same optimal value.

Comparative study

S.No.	Method	Initial Basic Feasible Solution	Optimal Solution (Stepping stone)
1	LCM	106.13	94.07
2	VAM	101.15	94.07
3	RAM	103.98	94.07

Conclusion

In this paper a Fuzzy Transportation Problem whose cost values are taken as Symmetric Triangular Fuzzy Numbers are considered. The Symmetric Triangular Fuzzy Numbers are transformed into crisp numbers using a proposed ranking function. The initial basic feasible solution and optimal solution is obtained by the usual VAM and Stepping Stone method respectively. The proposed method proves to be a better one from the comparative study.

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