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Research Article

SOME APPLICATIONS ON HOMOTOPY THEORY IN SCIENTIFIC FIELDS

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Abstract

We introduced the topological fundamental group and presented some interesting basic properties of the notion. Also we extend the notion to homotopy groups and try to prove some similar basic properties of the topological homotopy groups. We also study more about the topology of the topological homotopy groups order to find necessary and sufficient conditions for which the topology is discrete. We followed the analytical induction mathematical method and we found that studying homology groups may be more than cohomology fundamental groups.

Keywords: Topological Space, Continues, Homology, Homology Group, Graph Groups, Scalar Field, Homomorphism, Isomorphism, Random.

INTRODUCTION

Algebraic topology is twentieth century field of mathematics that can trace its origins and connection, and homology is one of the main idea of algebraic topology Algeria's topology is one of the most important creation in mathematics which uses algebraic tools to study topological spaces. The most important of this invariants are homology groups, homology groups and co- homology groups. The goal of this paper is to acquire uses, study some classes of algebraic topology (some underline geometry notation. The fundamental group homology ethology. Homogony theory and some application to scientific field.

The Topology of π_n^{top} (X)

We are going to study more on the topology of $\pi_n^{top}(X)$, specially we intend to find necessary and sufficient conditions for which$\pi_n^{top}(X)$is discrete.

Definition (1.1). A topological space X is called n-semilocally simply connected if for each $x \in X$ there exists an open neighborhood U of x for which any n-loop in Y is nullhomiotopic in X. In other words the induced homomorphism of the inclusion i_* : π_n (U, x) → π_n (X,x) is zero[1]

Theorem (1.2). If π_n^{top} is discrete, then X is n-semilocally simply connected.

Proof. For each x ∈X, sinceπ_n^{top}(X, r) is discrete, there exists an open neighborhood Win Hom((I^n, I^n), (X, x)) of the constant n-loop at x such that each element of W is homotopic to the constant loop at xBy compact-open topology, we can consider W as$\cap_{i=1}^{m} \langle K_i, U_i \rangle$, where K_i's are compact subsets of $I^n$and U_i's are open in X. Consider U = $\cap_{i=1}^{m}$ U_i as a nonempty open neighborhood, then $\langle I^n, U \rangle \subseteq$ W.

Therefore any n-loop in U at a belongs to W and so is null homotopic in x. Hence X is n-semilocally simply connected [3]. Note that the following examples show the inverse is not truc, in general. In both of them, we use the fact that the compact-open topology on Hom((I^2,i^2), (X,0)) is a equivalent to the uniformly convergence topology when X is metric space [6].

Example (1.3). Let X=U$U_{n\epsilon N} s_n$, where S_n

$$S_n=\left\{(x, y, z \middle| x - \tfrac{1}{4}\right\} \quad (1)$$

$$S_n=\left\{(x, y, z) \middle| x - \tfrac{n-1}{4})^2 + y^2 + z^2 = (\tfrac{n-1}{2n})^2\right\} \quad (2)$$

= {(x,y,z) | (S_1 = {(1, v.2)| (1 ² + y² + 2² = ^²=1 1² + 1² + 2² = (^2=¹)²), 211 for each n ≥2. Then (S) as a sequence of 2-loops in X at p= (0,0,0) uniformly

converges to S_1. Now (S_1) is a limit point in 2(X, z), nevertheless X is 2-Semilocally simply connected.

Example (1.4). Let X denotes the following subspace of R^3: 11 X= [0, 1]x [0, 1]x (0.1) U[0. 1] x (0, 1) x (0.1) U $\left\{0,1,\frac{1}{2},\frac{1}{3},\dots\right\}$X[0,1]X[0,1] (3)

Let p= (0,0,0). Consider the following sequence of 2-loops at p X_n= $[0,\frac{1}{n}]$x [0, 1]x {0.1}U $[0,\frac{I}{n}]$ x {0.1}x [0.1] U $\left\{\frac{1}{n}\right\}$X[0,1]X[0,1] (4)

Obviously, this sequence is uniformly convergent to the nullhomotopic loop Xo = (0) [0, 1]× [0, 1]. Thus 20(X) is not discrete, however one can see that X is 2-Semilocally simply connected.

Definition (1.5) Let f: X → Ybe a mapping of one topological space into another. A real function f (x), x ≥ 0 is said to be continuous at the point x_0 in x if for each neighborhood of f (x_0) there exists a neighborhood of x_0 Such that f(G) ⊆ H [1].

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Definition (1.6) Let XC R" and YcR". A function: $f: X \rightarrow Y$ is continuous if whenever yis an open set in Y, therf^{-1}(y) is an open set in X.

Definition (1.7) The function is continuous at the point x_o in Xif for each ε> 0 there exists δ>0 such that $d_1(x,x_0) < \delta \Longrightarrow d_2$. (f(x), $f(x_0)$) <ε [6].

Definition(1.8) Let f:X→Y be topological spaces and fand g continuous functions from X to Y. Then fis tiomotopic to g if there is a continuous family of continuous functions$f_1 X \rightarrow Y$ for $0 \leq t \leq 1$ satisfying:

i. $f_0 = f$
ii. $f_1 = g$
iii. $f_1(x)$is continuous both as a function of veX and as a function of [0.1]

Definition (1.9) For any topological space X, the abelian group D. and integer $n \leq 0$, there is sequence of Aeolian groups $H_n(X.D)$for $n = 0,1,2,3 \ldots \ldots \ldots \ldots \ldots$

Simplified Embedding's and Immersions

The containment problem for subcomplexes of random 2-dimensional complexes is similar to the containment problem for random graphs, we also study simplicial immersions, which are more general than simplicial embeddings. Let S be a 2-dimensional finite simplicial complex We assume that S is fixed. independent of n. The set of vertices of S is denoted by V(S).

Definition 2.1 A simplicial embedding g: S→ Y, where Y∈ G($\Delta_n^{(2)}$,p) is a random, 2-complex, is defined as an injective map of the set of vertice V(S) of Sinto the set of vertices $\{1, \ldots . n\}$ of Y satisfying the following condition: for any triple of distinct vertices (u_1), (u_2),(u_3)∈ V(S) which span a simplex in S, the corresponding points g(u_1), g(u_2), g(u_3) ∈ $\{1, \ldots . n\}$ span a face of Y [5].

The following definition describes a more general notion.

Definition 2.2 A simplicial immersion g: S Y into a random 2-complex Y ∈ G($\Delta_n^{(2)}$, P) is defined as a map of the set of vertices V(S) of 5 into the set of vertices $\{1, \ldots . n\}$ of Y satisfying the following two conditions

(a) for any triple of distinct vertices u_1, u_2, u_3∈ V (S) which span a 2-simplex in S. the corresponding points g(u_1). g(u_2), s(u_3) ∈ $\{1, \ldots . n\}$ are pairwise distinct and span a face of Y
(b) for any pair of distinct 2-simplexes and (σ) of (σ^1) S, the corresponding 2-simplexes g(σ) and g(σ^1) of Y are distinct [4].

Definition 2.3 For a simplicial 2-complex S let $_u$(S) denote

$$\mu(S) = \frac{u}{f} \in Q \quad (5)$$

where u =u_s and f = fs are the numbers of vertices and faces in S.

Definition 2.4 Let S be a finite 2-dimensional sinplicial complex. Define

$$\tilde{\mu}(S) = \min_{S' \subset S} \mu(S'), \quad (6)$$

where the minimum is formed over all subcomplexes S' ⊂S or. equivalently, over all pure subcomplexes$S' \subset$ SNote that the invariantis$\tilde{\mu}$ monotone decreasing: if S is a subcomplex of 'T then$\tilde{\mu}$(S) ≥ $\tilde{\mu}$(T) [2].

Balanced and unbalanced triangulations

Definition 2.5 A finite simplicial 2-complex S is called balanced if (S) = μ(S)= $\tilde{\mu}$(S) i.e. if the quantities defined in Definitions 2.7 and 2.8 coincide. In other words, S is balanced if μ(S) ≤ μ (S′) for any subcomplex (S′)′ ⊂ S [4].

Definition 2.6 is similar to the corresponding notion for random graphs, see

Example 2.7 Let S = Σ_g be a triangulated closed orientable surface of genus $g \geq 0$. Then x(S)=2-2g=u-e+f where u, e, f denote the numbers of vertices, edges and faces in S correspondingly. Each edge is contained in two faces whichgives 3f = 2e and therefore

$$\mu(\Sigma_g) = \frac{1}{2} + \frac{2-2g}{f} \quad (6)$$

Similarly, if S = N_g is a triangulated closed nonorientable surface of genus $g \geq 1$ then x(N_g) =2-g and

$$\mu(N_g) = \frac{1}{2} + \frac{2-g}{f} \quad (7)$$

Formulae (2) and (3) give the following:

Corollary 2.8 The invariant μ(Σ)of an orientable triangulated surface Σ_g sut isfies:

(1) 1/2 <Σ_g ≤1 for g=0 (since $f \geq 4$);
(ii) (Σ_g) = 1/2 for g=1 (the torus);
(iii) (Σ_g) < 1/2 for g> 1;
(iv) Iff→∞(i.e. when the surface is subsequently subdivided) then u(Σ_g) → 1/2. Corollary[5]

Corollary 2.9 The invariant u(N_g) of a nonorientable triangulated surface N_g satisfies:

(i) 1/2 <μ(N_g) ≤ 3/5 for g=1 (since f≥10);
(ii) μ(N_g) =1/2 for g=2 (the Klein bottle);
(iii) μ(N_2) < 1/2 for g> 2;
(iv) Iff → ∞ (i.e. when the surface is subsequently subdivided) then (N_g) ⟶ 1/2 [6].

Example 2.10 Let S be a triangulated disc. Then x(S) = u- e+ f = 1 and 3f = 2e – e_0 to where co is the number of edges in the boundary OS. Substituting = (3f+e_0)/2, one obtains

$$\mu(S) = \frac{1}{2} + \frac{e_0}{2f} + \frac{1}{f^*} \quad (8)$$

As a specific example consider the regular n-gon S shown on the figure on the left. Then $u = n + 1, f = n, e_0 = n$=and

$$\mu(s) = 1 + \frac{1}{n^*} \quad (9)$$

Quantizing the Free Relativistic Scalar Field

Our goal is to quantize the free relativistic scalar field in n spatial dimensions. (Note that we will use a more naturally relativistic notation than Peskin and Schroeder [7], although we will keep the particle physicists mostly minus convention for the metric).

We will postulate that the relevant relative is tically invariant Lagrange (density) is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_0)^2 - \frac{1}{2} m_0^2 \phi_0^2 \quad (10)$$

We seek a solution do associated with the above Lagrange (density) and a spectrum of the field operator (eigenvalues and eigenvectors). We will find that do is the bare (renormalized) operator associated with the production of a single scalar particle of mass mo. For notational simplicity, for the rest of this section we will drop the 0 subscript and the operator hat. (Note that since the theory is non-interacting, we can solve the theory exactly and without renormalization.) Dimensional analysis is an invaluable tool for the physicist, and so we should pause for a moment to analyze the dimensions of the objects in our theory. The path integral goes as J Does, where SJ dz +1C. Since one can only take a number to the power of a dimensionless number, the action must be dimensionless, [S] = 1. Since [+¹] = "t¹ = E(n+1), we have that [C] =E+¹ Since [0]=[m] = E, we have that [] - E-11/2 Our solution must first of all satisfy the classical equations of motion found by extremizing the action. The Euler-Lagrange equations yield.

$$(E + m^2)\phi = 0 \quad (11)$$

Already things are looking very promising: the equations of motion are Lorentz invariant (as they must be as the Lagrangian is Lorentz invariant). We may readily solve Eq. 3 by decomposing our solution into Fourier modes.

$$\phi(x) = \int \frac{b^n}{(2\pi)^n 2E_{\vec{p}}} \left(\tilde{a}_{\vec{p}} e^{ipx} + \tilde{a}_{\vec{p}} e^{ipx}\right) p^0 = E_{\vec{p}} \sqrt{p^{-2} + m^2} \quad (12)$$

Notice how: 1) we have (explicitly) separated out the classical from the quantum in Eq. 4, where $\hat{\phi} = \hat{\phi}$automatically; 2) the Fourier modes obey the usual relativistic dispersion relation; and 3) dimensional analysis implies that $E^{(n-1)/2} = E^{n-}[\hat{a}]$, so then $[\hat{a}] = E^{(1-n/2)} = [\hat{a}t]$. In order to quantize Eq. 10, we must impose the Dirac quantization condition. We must of course then first decide what the Dirac quantization condition is. Since we are interested in relativistic theories, we will require that fields cannot influence each other outside of the lightcone. Hence a sensible generalization from the 1D NRQM case of$\hat{x}, \hat{p}$ =i for the Dirac quantization condition for fields is to require an equal-time contact interaction:

$$[\phi(x^0, \vec{x})\hat{\pi}(\vec{y}] = is^n(\vec{x} - \vec{y}) \quad (13)$$

Le fields at the same time (in one inertial frame) may only affect each other at exactly equal points (otherwise information could propagate faster than the speed of light). As an aside, it's an interesting exercise, once we've solved for $\phi(x)$, to compute the commutator $[\hat{\phi}(x), \hat{\phi}(y),]$ without the equal time restriction! [3].

Theorem 3.1 Fix an integer r > 1 and consider the r-thBetti number of the associated graph group,

$$b_e: G(\Delta_n^{(1)}, p) \rightarrow z, b_r(T) = b_r(A_r) \quad (14)$$

as a random function of a random graph. If the limit (8) erists and is positive then for any integer k= 0, 1,..., the probability P $(b_r\ (A_r) = k)$ converges (as n→∞)

to $e^{-\lambda} \frac{\lambda^k}{k!}$ where = $\lambda = \frac{c^r}{r!}$

In other words, the limiting distribution is Poisson with meanλ [5].

Example 3.2. Let X = $\cup_{n \in N} S_n$ N, where

$$S_1 = \left\{(x, y, z) \middle| (x - \tfrac{1}{2})^2 + y^2 + z^2 = \tfrac{1}{4}\right\}, \quad (15)$$

$$S_n = \left\{(x, y, z) \middle| (x - \tfrac{n-1}{2n})^2 + y^2 + z^2 = (\tfrac{n-1}{2n})\right\}. \quad (16)$$

for each n ≥2. Then $\{S_n\}$as a sequence of 2-loops in X at p= (0,0,0) uniformly converges to S. Now $(S_1]$ is a limit point in π_2^{top} (X, x), nevertheless X is 2- semilocally simply connected.

Example 3.3. Let X denotes the following subspace of R^3

$$x = [0,1] \times [0,1] \times \{0,1\} \cup [0,1] \times \{0,1\} \times [0,1] \cup \left\{0, 1, \tfrac{1}{2}, \tfrac{1}{3} \ldots\right\} \times [0,1] \times [0,1]\,. \quad (17)$$

Let p= (0,0,0). Consider the following sequence of 2-loops at p

$$X_n = \left[0, \tfrac{1}{n}\right] \times [0,1] \times \{0,1\} \cup \left[0, \tfrac{1}{n}\right] \times \{0,1\} \times [0,1] \cup \left\{\tfrac{1}{n}\right\} \times [0,1] \times [0,1] \quad (18)$$

Obviously, this sequence is uniformly convergent to the nullhomotopic loop X_0 = {0}x [0, 1] x [0, 1]. Thus π_2^{top} (X) is not discrete, however one can see that X is 2- semilocally simply connected [7].

The Struggle for Scientific Authority

A particular kind of social capital which gives power over the constitutive mechanisms of the field, and can be reconverted into other forms of capital, owes its specificity to the fact that the producers tend to have no possible clients other than their competitors (and the greater the autonomy of the field, the more this is so). This means that in a highly autonomous scientific field, a particular producer cannot expect recognition of the value of his products ("reputation", "prestige". "authority", "competence", etc.) from anyone except other producers, who, being his competitors too, are those least inclined to grant recognition without discussion and scrutiny. This is true de facto: only scientists involved in the area have the means of symbolically appropriating his work and assessing its merits. And it is also true de jure; the scientist who appeals to an author ity outside the field cannot fail to incur discredit (In this respect, the scientific field functions in exactly the same way as a highly autonomous artistic field": one of the principles of the specificity of the scientific field lies in the fact that the competitors must do more than simply distinguish themselves from their already recognised precursors: if they are not to be left behind and "outclassed", they must integrate their predecessors and rivals work into the distinct and distinctive construction which transcends it.). In the struggle in which every agent must engage in order to force recog. nition of the value of his products and his own authority as a legitimate producer, what is at stake is in fact the power to impose the definition of science (i.e. the delimitation of the field of the problems, methods and theories that may be regarded as scientific) best suited to his specific interests, i.e. the definition most likely to enable him to occupy the dominant position in full legitimacy, by attributing the highest position in the hierarchy of scientific values to the scientific capacities which he personally or institutionally possesses (e.g. by being highly trained in mathematics, having studied at a particular educational institution, being a member of a particular scientific institution, etc.). In more than one debate on the priority of a scientific discovery, the scientist who discovered the unknown phenomenon, often in the form of a simple anomaly not covered by existing theories, has clashed with the scientist who made a new scientific fact of it by set ting it in a theoretical construction irreducible to the simple empirical datum. These political arguments about scientific property rights, which are at the same time scientific debates on the meaning of what has been discovered and epistemological arguments as to the nature of scientific discovery, are in reality the expression of the conflict between two principles.

The Hemotopy of the Agring Process

The study of topology relies on the intrinsic properties of the body. It is coordinate-free since it does not depend on theproperties of the chosen coordinates of body. In the study the human body xis consideredin the topological space since our goal is to study the continuous function of the ageing process. The importance of it all is to construct the algebraic invariants such as homotopy that reflect the connectivity of the body. Consider the human body

$x = s^1 x1$ SI and let $x \in x$ and $t \in T$ define the growth of the body and the age of the body respectively. Since the final age of the human body is not known let $t = \phi$ represent the final age of the body such that $t \in [\beta, \infty]$ denotes the age interval of the body from t=β, to t=∞. The time t=∞. is the age threshold value of the human body. The ageing process for all tet $\in$ T is the family or the sequence of the functions $f_1(x)$ which occurs as $f_1(x)$ approaches g(x)$\in \infty$ for all g(x). Suppose $t \in [0, \infty]$ is the interval of the ageing body and $f.g: x \to x$ are the two functions of the topological shape of the human body,then the shape of the wrinkled body ∞ years old is given by the continuous map $f_1(x)$ such that $f_0(x) = f(x)$ and $f_\infty(x) = g(x)$ as shown in Figure 1.

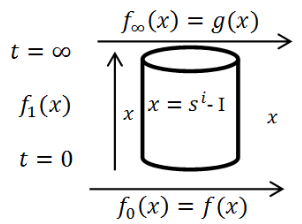


Figure 1. The shape of a wrinkled human body from f (x) to g(r)

The closed connected human body x S x/ is defined by the functions $f.h: x \to x$ on the interval $1 = [0, \beta]$ where fandhare homotopy equivalences. The interval = $1[0, \beta]$ provides the initial age and height of the whole body $x = s^1 \times 1$ [3].

Homology

We study homological properties of finite free complexes over noetherian local hinges. The result uncover novel links between the structure of the homology modules of such complexes and cornmeal modules of the ring. The arguments use techniques from commutative algebra, differential graded homological algebra [2].

Lemma (6.1):

For every complex of B- module M there is an in equality

$$\text{Level}_R^C(m) \leq \inf[\Sigma \text{Level}_R^C(l_n)] \; L = \text{mindc}(R) \qquad (19)$$

Proof:

$$\text{As Level}_R^C = \text{Level}_R^C(l) \qquad (20)$$

It suffices to assume that only finitely many components have non-zero and to show that then one has

$$\text{Level}_R^C(l)\Sigma_{n6z}\text{Level}_R^C(l_n) \qquad (21)$$

The complex L admire filtration

$L \leq n - 1\, c^l c \ldots\ldots.,$

And there are isomorphism

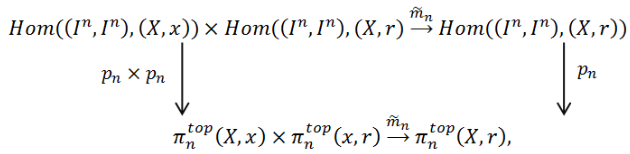
$$L \leq n \,/\, L \leq n - 1 = \Sigma^n l_n \qquad (22)$$

Yield's

$$\text{Level}_R^C(\Sigma^n l_n) = \text{Level}_R^C(l_n) \qquad (23)$$

Theorem (6.2). Let(x_x) be a pointed space. Thenπ_n^{top} is a topological group for all n≥ 1.

Proof. In order to show that the multiplication is continuous, we consider the following commutative diagram



where $\widetilde{m}_m$ is concatenation of n-loops, and m,, is the multiplication in $\pi_n^{top}(X, x)$

Since $(p_n \times p_n)$-¹$m_m - 1$ ¹ (U) = $\widetilde{m}_n^{-1}(U)$ for every open subset U of $\pi_n^{top}(X, x)$, it is enough to show that m, is continuous. Let $\langle K, U \rangle$ be a basis ment in

$\text{Hom}((I^n, I^n), (X.x))$Put

$$K_1 = \{(t_{1,\ldots\ldots}t_n)\left(t_1 \ldots\ldots, t_{n-1}, \frac{t_n}{2}\right) \in K\} \qquad (24)$$

And

$$K_2 = \{(t_{1,\ldots\ldots}t_n)\left(t_1 \ldots\ldots, t_{n-1}, \frac{t_n+1}{2}\right) \in K\} \qquad (24)$$

Then

$$\widetilde{m}_n^{-1}\left\langle\langle K.U\rangle\right\rangle = \{(f_1, f_2)\, f_{1,} * f_{2,}(K) \subseteq\} = \langle K_2.U\rangle \times \langle K_2.U\rangle \quad (25)$$

is open in Hom $((I^n, I^n), (X.x))$ x Hom $((I^n, I^n), (X.x))$and so $\widetilde{m}_m$ is continuous.

To prove that the operation of taking inverse is continuous, let K be any compact subset of I^{n}" and put

$$K^1 = \{(t_1 \dots \dots ., t_n)\ (t_1 \dots \dots, t_n) \epsilon K\} \quad (26)$$

Clearly an t-loop a is in $\cap_{i=1}^{m} \langle k_i U_i \rangle$ if and only if its inverse is in $\cap_{i=1}^{m} \langle K^{-1} U_i \rangle$ where $\langle k_i U_i \rangle$are basis elements in Hom((I^n, I^n), (X. x)). Hence the inverse map is continuous. From now on, when we are dealing with to, by the notion we mean the isomorphism in the sense of topological groups. The following result shows that the topologital group is independent of the base point z in the path component [1].

Cohomological Dimension of Random Graph Groups

The cohomological dimension of A_r equals the size of the maximal clique in T. Recall that a clique in a graph is defined as a maximal complete subgraph. Theclique number cl(T) of a graph T is the maximal order of a clique in T. There are many results in the literature about the clique number of random graphs; we may interpret these results as statements about the cohomological di mension of graph groups build from random graphs, discovered that for fixed values of p the distribution of the clique number a random graph is highly concentrated in the sense that almost all random graphs have about the same clique number. These results were developed further by Bollobás and Erdős; see the monographs of B. Ballobás [4] and of N. Alon and J. Spencer [2]. We restate a result of a statement about cohomological dimension of random graph groups. Denote

$$z(n, o) = 2\log_q n - 2\log_q n + 2\log_q (e/2) + 1 \quad (27)$$

where q = p^{-1}. We assume that p is independent of n.

Theorem 7.1 Fir an arbitrary e > 0. Then

$$\lfloor z(n.p) - \in \rfloor \leq cd(A_r) \leq \lfloor z(n.p) + \in \rfloor \quad (28)$$

asymptotically almost surely (a.a.s). In other words, the probability that a graph T∈G ($\Delta_n^{(1)}$, p) does not satisfy inequality (10) tends to zero when n tends to infinity. Here$\lfloor x \rfloor$ denotes the largest integer not exceeding r. We may assume that $\in < 1/2$; then the integers $\lfloor z(n,p) - \in \rfloor$ and $\lfloor z(n,p) + \in \rfloor$ either coincide or differ by Thus, according to Theorem 3.2, the cohomological dimension cd(A_r) for a random graph I takes on one of at most two values depending on n. and p, with probability approaching 1 as $n \to \infty$ [4].

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