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Research Article

FUZZY LOGIC STRATEGY IMPLEMENTED ON AN OPTIMAL CONTROL PROBLEM OF HYPOXEMIC HYPOXIA TISSUE BLOOD DIOXIDE CARBON EXCHANGE: CARDIOVASCULAR-RESPIRATORY SYSTEM RESPONSE TO EXERCISE

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Abstract

In this paper, we present an approach integrating fuzzy logic strategy for solving an optimal control problem of hypoxemic hypoxia tissue blood dioxide carbon exchange in human body. The response to exercise is high- lighted by existence of heart rate and alveolar ventilation which are controls of cardiovascular-respiratory system. In order to verify the adequacy of fuzzy logic strategy, numerical results for a 30 years old woman performing jogging are compared with ones obtained direct method. The findings are in good agreement with empirical data. Therefore approach integrating the fuzzy logic strategy is very important for solving optimal control problem and it induces the performance of cardiovascular-respiratory system.

Mathematics Subject Classification : 92C30, 49J15
Keywords: Fuzzy logic, Optimal control, Membership, Hypoxia, Pressure, Gas, Numerical simulation.

INTRODUCTION

Human body can be deprived of adequate oxygen supply at the tissue level. This pathological condition refers to hypoxia can affect the whole body (Anoxia), or a region of the body (Tissue hypoxia) [1]. Hypoxemic hypoxia refers specifically to hypoxic states where the arterial content of oxygen is insufficient [2]. The insuffi- cient oxygen delivery (hypoxemia) is due to low of partial arterial pressure of oxygen (PaO_2) or inability to use it [3], [4], [5] and high partial arterial pressure of dioxide carbone ($PaCO_2$). Moreover, human body is facing with hypoxia when PaO_2 is less than $55mmHg$ and $PaCO_2$ greater than $45mmHg$. This change of pressures of cardiovascular-respiratory system controlled by central chemoreceptors in the hy-pothalamus [6] causes ventilation to decrease [7]. Anoxia or tissue hypoxia may not be present only in patients since it occurs when healthy people ascend to high alti- tude or when breathing mixtures of gases with a low oxygen content. The hypoxia happens also during physical activity. Moreover, human body needs more oxygen during physical activity the muscles uses oxygen amounts available in the blood. This mechanism is amplified when efforts involving large muscle groups such as cy- cling or running that are performed in hypoxia [8], [9]. Therefore, physical activity plays a crucial role if deficiency in the amount of oxygen reaches the tissues of the body that is when hypoxemic hypoxia happens. Mathematically, this mechanism can be done by solving an optimal control problem of cardiovascular-respiratory system where heart rate and alveolar ventilation consist of controls. The objective of this paper aims at using an approach integrating the fuzzy logic strategy to solve this optimal control problem. The techniques based on fuzzy logic control system proposed by Lotfi Zadeh [10] in 1965 are methods which imitate human's decision making [11].

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Fuzzy logic control system based on analyzes analog input values into logical variables through mathematical system with continuous values between 0 and 1. It is used by scientists since it can be implemented in hardware, software, or a combination of both. Fuzzy logic system uses the sets that are represented by a membership function which defined on the universe of discourse and maps each elements of universe throw numerical values in the interval [0, 1]. This paper is organised as follows. Section 2 presents the methods and material. An optimal control problem is defined presented in this section. The section 3 deals with discrete form of optimal control problem and model equations using both fuzzy logic strategy and direct method. The numerical simulation is presented in section 4 while section 5 focuses on concluding remarks.

METHODS AND MATERIALS

We consider the mathematical model of mass transport of dioxide carbon exchange in tissue developed by Guillermo Gutierrez [12]. The diagram of this mathematical model is illustrated in the Figure 1. The equations models are as follows.

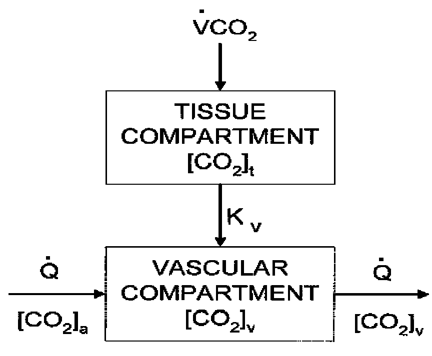

Figure 1. Diagram for the tissue dioxide carbon exchange model

$$\frac{d\,[CO_2]_t}{dt} = \dot{V}CO_2 - Kv([CO_2]_t - [CO_2]_v), \quad (1)$$

$$\frac{d\,[CO_2]_v}{dt} = \dot{Q}([CO_2]_a - [CO_2]_v) + Kv([CO_2]_t - [CO_2]_v), \quad (2)$$

where $[CO_2]$ denotes the total dioxide carbon exchange concentration (dissolved and bound). The subscripts t and v refer to the tissue and vascular compartments respectively. The parameter Kv is the mass transfer coefficient for CO_2. The CO_2 production is $\dot{V}\ CO_2$ and the rate of change of $[CO_2]_v$ depends on blood flow per unit volume of tissue ($\dot{Q}$) in vascular compartment. The controls of respiratory and cardiovascular are systems are alveolar ventilation ($\dot{V}A$) and heart rate (H), respectively

While a person performing physical activity, respiratory control system varies the ventilation rate in response to the levels of CO_2 and O_2 in the body and the controlmechanisms of cardiovascular system influences global control in the blood vessels as well as heart rate for changing blood flow $\dot{Q}$[13]. Generally, this mechanism of control happens in altitude and particular in the hypoxia case. Mathematically, the role of these controls is modeled by the following equations

$$\frac{dH}{dt} = u(t), \quad (3)$$

$$\frac{d\dot{V}_A}{dt} = v(t), \quad (4)$$

where $u(t)$ and $v(t)$ are functions determined by an optimality criterion.

Let consider

$$A = \exp\left[0.385 \times \ln(\frac{SvO_2}{1 - SvO_2}) + 3.321 - (72 \times SvO_2) - \frac{[SvO_2]^6}{6}\right] \quad (5)$$

and

$$B = \frac{[CO_2]_v \times ((2.244 - 0.422 \times SO_2) \times (8.74 - pH))}{((2.244 - 0.422 \times SO_2) \times (8.74 - pH)) - 0.43}. \quad (6)$$

In (5) the quantity $SvO2$ is venous oxyhemoglobin saturation and it is defined as follows

$$SvO_2 = \frac{[O_2]_v}{1.34 \times [Hb]}.$$

The expression (6) refers to content of plasma defined by Douglas [14] where SO_2 denotes sulfur dioxide, pH is constant which shows how strongly acid is and where $[Hb]$ refers to the blood hemoglobin concentration in g/L and in numerical simulationwe consider that it is equal to 7.40g/L. Using Fick's law, the venous concentration ($[O_2]_v$) in the vascular compartment is expressed as follows

$$[O_2]_v = [O_2]_a - \frac{\dot{V}O_2}{\dot{Q}},$$

where $[O_2]_a$ and $\dot{V}\ O_2$ denote arterial concentration of oxygen and production of oxygen respectively. The partial arterial pressure of oxygen is given by (See [12] and[15] for details)

$$PaO_2 = A \times \exp[2.016(7.4 - pH)]. \quad (7)$$

and Henderson-Hasselbach equation [16] leads to the following equation of partial arterial pressure of oxygen dioxide carbon

$$PaCO_2 = \frac{B}{0.06868 \times [10^{(1.04214pH - 6.41036)}]}. \quad (8)$$

Let P^eaO_2 and P^eaCO_2 be equilibrium value of PaO_2 and $PaCO_2$ respectively. Since respiratory control system aims at keeping arterial pressures closely to their equilibrium values, optimal control problem is formulated as follows.

Define

$$J(u,v) = \int_0^{T_{\max}} q_{CO_2}\left(PaCO_2 - P^eaO_2\right)^2 + q_{O_2}\left(PaO_2 - P^eaCO_2\right)^2 + q_u u(t)^2 + q_v v(t)^2, \quad (9)$$

solve the following coefficient identification problem.

Find control $U^* = (u^*, v^*)^t$ such that

$$U^* = \arg \min_{U=(u,v)^t} J(U), \quad (10)$$

subject to the model equations (1)-(4) where $q_{CO}2$, q_O2 , q_u and q_v are weight con- stants while in (9) $T_{\max}$ denotes the maximum time the physical activity is per- formed.

DISCRETE OPTIMAL CONTROL PROBLEM

The cost function (9) and model equations (1)-(4) are discretized using two different methods: fuzzy logic strategy and direct method. Fuzzy logic strategy is developed by Takagi-Sugeno [17], [18]. For more details, refer also to [19], [20] and [21]. The use of two different methods leads to compare the solutions in order to test efficiencyof fuzzy logic strategy.

We start the discretization using fuzzy logic strategy. Let consider

$$\alpha = RQ \times SV \times \left([O_2]_a - [O_2]_v\right), \quad (11)$$

and

$$\beta = \dot{Q} \times K_{CO_2} \times \dot{V}CO_2K. \quad (12)$$

The model equations (1)-(4) becomes

$$\begin{cases} \frac{d\,[CO_2]_t}{dt} = \alpha H - Kv\left([CO_2]_t - [CO_2]_v\right), \\ \frac{d\,[CO_2]_v}{dt} = Kv[CO_2]_t + \frac{\beta}{\dot{V}_A} - (\dot{Q} + Kv)[CO_2]_v + \dot{Q}k_{CO_2}, \\ \frac{dH}{dt} = u(t), \\ \frac{dV_A}{dt} = v(t). \end{cases} \quad (13)$$

Hence, using explicit Euler's method we get

$$\begin{cases} [CO_2]_t^{k+1} = [CO_2]_t^k + h(-Kv\,[CO_2]_t^k + Kv\,[CO_2]_v^k + \alpha H^k), \\ [CO_2]_v^{k+1} = [CO_2]_v^k + h(Kv\,[CO_2]_t^k - (\dot{Q} + Kv)\,[CO_2]_v^k + \dfrac{\beta}{\dot{V}_A^k} + \dot{Q}k_{CO_2}), \\ H^{k+1} = H^k + h(u(t)), \\ \dot{V}_A^{k+1} = \dot{V}_A^k + h(v(t)), \end{cases} \quad (14)$$

where $h = \dfrac{T_{\max}}{N}$.

Let $X = ([CO_2]t\,,[CO_2]v\,,H, \dot{V}A)^t$ be operating point and assuming $s = 1, 2, 3$ be operating point number associated X we have the system

$$X_s^{k+1} = A_s X_{k,s} + B_s U_{k,s} + C_s, \quad s = 1,2,3, \quad (15)$$

Where

$$A_s = \begin{pmatrix} 1 - hKv & hKv & \alpha h & 0 \\ hKv & 1 - h(\dot{Q} + Kv) & 0 & 0 \\ 0 & 0 & h & 0 \\ 0 & 0 & 0 & h \end{pmatrix},$$

$$B_s = h\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \text{ and } C_s = h\begin{pmatrix} 0 \\ \dot{Q}k_{CO_2} + \dfrac{\beta}{\dot{V}_{A,s}^k} \\ 0 \\ 0 \end{pmatrix}. \quad (16)$$

Let consider

$$Y = (PaCO_2 - P^e aCO_2, , PaO_2 - P^e aO_2)^t, \quad (17)$$

thus using rectangular method approximation of cost function (9) becomes

$$J(Y,U) = \sum_{k=0}^{N-1} (Y_k^T R Y_k + U_k^T B U_k)h, \quad (18)$$

where

$$R = \begin{pmatrix} q_{CO_2} & 0 \\ 0 & q_{O_2} \end{pmatrix}, \qquad B = \begin{pmatrix} q_u & 0 \\ 0 & q_v \end{pmatrix}. \quad (19)$$

Finally, the optimal control problem (10) subject to (1)-(4) becomes the a linear quadratic (LQ) problem formulated as follows.

Find $U* = (U0*; :::; UN*-1)t$ solution of

$$\min_U J(Y,U) = \sum_{k=0}^{N-1} (Y_k^t R Y_k + U_k^t B U_k)h, \quad (20)$$

subject to

$$X_s^{k+1} = A_s X_{k,s} + B_s U_{k,s} + C_s, \quad s = 1,2,3. \quad (21)$$

We now focus on the discretization of the model equations (1)-(4) and cost function using direct method. We consider linear B-splines functions

$$\mathcal{B}^N = \left\{\psi_j^N,\ j = 1,...,N\right\}, \quad (22)$$

defined on the uniform grid

$$\Omega_N = \left\{t_k = \frac{kT_{\max}}{N},\quad k = 0,...,N\right\}. \quad (23)$$

such that

$$\psi_j^N(t_k) = \delta_{ik},$$

where δ denotes Kronecker symbol. Hence, using B-splines functions the discrete equations model of the system (1)-(4) is as follows.

$$\frac{d\,[CO_2]_t^N}{dt} = \dot{V}CO_2 - Kv([CO_2]_t^N - [CO_2]_v^N), \quad (24)$$

$$\frac{d\,[CO_2]_v^N}{dt} = \dot{Q}([CO_2]_a - [CO_2]_v^N) + Kv([CO_2]_t^N - [CO_2]_v^N), \quad (25)$$

$$\frac{dH^N}{dt} = u^N(t), \quad (26)$$

$$\frac{d\dot{V}_A^N}{dt} = v^N(t), \quad (27)$$

$$[CO_2]_t^N(0) = [CO_2]_t^{N,0}, \quad [CO_2]_v^N(0) = [CO_2]_v^{N,0}, \quad (28)$$

such that

$$\left|[CO_2]_{t,0} - [CO_2]_t^{N,0}\right| \underset{N\to\infty}{\longrightarrow} 0, \quad \left|[CO_2]_{v,0} - [CO_2]_v^{N,0}\right| \underset{N\to\infty}{\longrightarrow} 0,$$

and

$$\left|H_0 - H^{N,0}\right| \underset{N\to\infty}{\longrightarrow} 0, \quad \left|\dot{V}_{A,0} - \dot{V}_A^{N,0}\right| \underset{N\to\infty}{\longrightarrow} 0.$$

Let set
$x = (PaCO_2, PaO_2)^t$ and $x^e = (P^e aCO_2, P^e aO_2)^t$.

The discretization of the cost function (9) in ΩN can be written as follows

$$J^N(U^N) = \sum_{k=1}^{N}\left(\sum_{i=1}^{2} k_i\left(x_i^N(t_k) - x_i^e\right)^2 + \sum_{j=1}^{2} q_j\left(U_{j,k}^N\right)^2\right)h. \quad (29)$$

where

$$k = (q_{CO_2}, q_{O_2})^t \text{ and } q = (q_u, q_v)^t.$$

The $x_i,\ x_i^e,\ U_i,\ k_i$ and qi are the ith component of the vectors x; x^e; U; k and q respectively.

Let consider $V = C^0(0; T\max)$, we want to find

$$U^N = (U_1^N, U_2^N) \in Q^N$$

that minimizes the cost function (29) in $Q^N = (V^N)^2$ such that

$$U_j^N = \sum_{k=0}^{N} U_{j,k}^N \psi_k(t), \quad j = 1,2. \quad (30)$$

Finally using B-splines functions, the discrete formulation of optimal problem (10) subject to (1)-(4) is written as follows.

$$\min_{U^N \in \mathbb{R}^{(N+1)} \times \mathbb{R}^{(N+1)}} J^N(\underline{U}^N) \approx \Delta t\left((x^t_{i,e} R x_{i,e}) + (\underline{U}^N)^t B \underline{U}^N \right) \quad (31)$$

subject to (24)-(28) where $\underline{U}^N$ is a matrix $(N+1) \times 2$ such that the components $U^N_{j,k}$ are components of the function U^N_j in the set B^N and $x_{i,e}$ denotes the matrix with $(i, k)^{th}$ component $x^N_i(t_k) - x^e_i$, R and B are matrices as in (19).

NUMERICAL SIMULATION

The numerical tests deal with a 30 years old woman performing three physical activity: walking, jogging and running fast for a period of $T_{\max} =$ 10 minutes. The Table 1 shows the values of her determinant parameters [22]. We consider

Table 1. The values of determinant parameters of cardiovascular-respiratory systemat equilibrium state for woman 30 years old who performs three physical activities

Exercise intensity	Rest	Walking	Jogging	Running Fast
Ventilation (L/min)	6	8.5	15	25
Heart rate (Beats /min)	70	85	140	180
Arterial Pas(mmHg)	140	110	135	170
Venous Pvs(mmHg)	3.566	3.46	3.28	3.23

$P^e aCO_2 = 38mmHg$, $P^e aO_2 = 95mmHg$ and $N = 100$ and values of parametersare given in Table 2 .

Table 2. Values of parameters used in the mathematical model cost function

Parameter	Value	Parameter	Value
$\dot{Q}$	6	$P^e a_{co_2}$	38
Kv	0.05	$P^e a_{o_2}$	95
pH	7.35	$(CO_2)^e_t$	26.5
SaO_2	0.98	$(CO_2)^e_v$	18.5
$\dot{V}CO_2$	0.21	P_{H_2O}	47
P_{ATM}	760	RQ	0.8
F_{IO_2}	0.21	SV	0.7
q_{co_2}	10	K_{CO_2}	0.0065
q_{o_2}	15	k_{CO_2}	0.244
q_H	100	$[O_2]_a$	0.197
q_V	100	$[O_2]_v$	0.147
K	863		

According to physiology of cardiovascular-respiratory system, the bound of each model variable is taken as shown in the Table 3.

Table 3. Bound of each model variable

Model variable	Interval
$[CO_2]_t$	$[25, 35]$
$[CO_2]_v$	$[15, 30]$
H	$[50, 180]$
$\dot{V}_A$	$[4, 25]$

Four linguistic variables of discourse X defined using variables of mathematical model (1)-(4) are as follows.

- CO2t: total tissue CO_2 concentration
- CO2v: total vascular CO_2 concentration,
- H: heart rate
- V_A: ventilation rate.

Since operating points take the corresponding values in the labels centers of a universe of discourse X [23], using bound of model variables, fuzzy terms as well as centered values for minimum, middle and maximum of labels are given in Table 4.

Table 4. Fuzzy terms and value of centered fuzzy label

Linguistic variable	Fuzzy term	Centered label value
CO2t	CO2tmin	25
	CO2tmiddle	30
	CO2tmax	35
CO2v	CO2vmin	15,
	CO2vmiddle	22.5
	CO2vmax	30
H	Hmin	50,
	Hmiddle	115
	Hmax	180
V_A	$V_A$min	4
	$V_A$middle	14.5
	$V_A$max	25

The operating points associated to above linguistic variables are given in the Table 5 while membership functions corresponding to this labeling are illustrated in the Figure 2 and 3. The Table 6 shows the obtained degrees of membership of each linguistic variable.

Table 5. Variables and their operating points

Variable	Operating Points
$[CO_2]_t$	$[25; 30; 35]$
$[CO_2]_v$	$[15; 22.5; 30]$
H	$[50; 115; 180]$
$\dot{V}_A$	$[4; 14.5; 25]$

Table 6. Variables and their corresponding membership degrees

Variable	w_{1f}	w_{2f}	w_{3f}
$[CO_2]_t$	0	0.4	0.6
$[CO_2]_V$	0	0.27	0.73
H	0.3	0.7	0
V_A	0.38	0.62	0

Using parameters values presented in the Table 2 the relation (11) and (12) lead

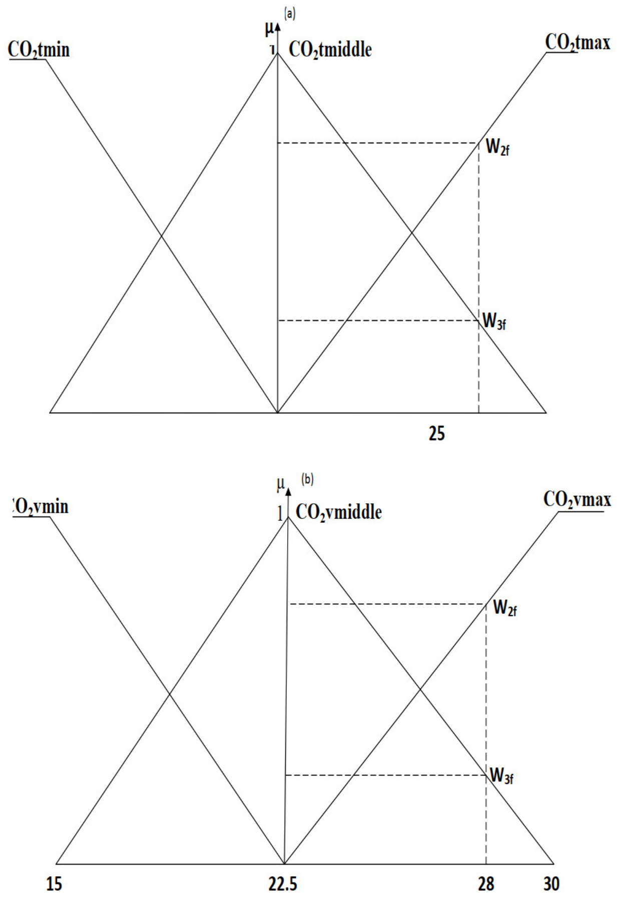

Figure 2. Membership function of $\Pi^{\cdot}_{\cdot 2\ldots}$ (a) and $\Pi^{\cdot}_{\cdot 2\ldots}$ (b)

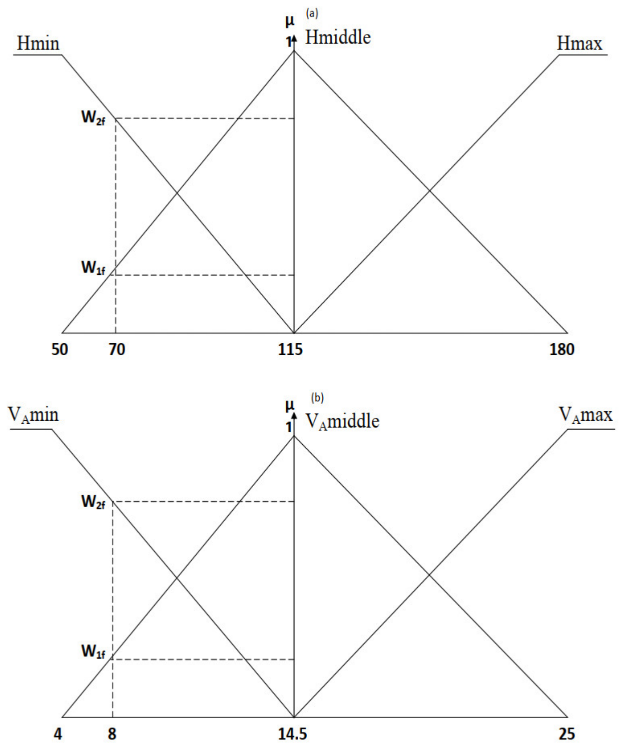

Figure 3. Membership function of □ (a) and □ (b)

to $\alpha = 0{:}028$ and $\beta = 7{:}068$; respectively and after calculations we get the following matrices which remain unchanged.

$$A_s = \begin{pmatrix} 0.995 & 0.005 & 0.0028 & 0 \\ 0.005 & 0.395 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \; B_s = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \end{pmatrix}, s = 1,2,3$$

$$C_1 = \begin{pmatrix} 0 \\ 0.3231 \\ 0 \\ 0 \end{pmatrix}, \; C_2 = \begin{pmatrix} 0 \\ 0.1951 \\ 0 \\ 0 \end{pmatrix}, \; C_3 = \begin{pmatrix} 0 \\ 0.1747 \\ 0 \\ 0 \end{pmatrix}.$$

Therefore, the optimal control problem (20)-(21) becomes a linear quadratic problem and using the theory of feedback control we get its solution of the form [24]

$$U_{i,k} = -K_i x_k, i = 1, ..., S; \quad k = 0, ..., N-1, \tag{32}$$

where

$$K_i = (Q + B_i^T E_i B_i)^{-1} B_i^T E_i A_i,$$

is the feedback gain matrix and *Ei* discreet Riccati equation solution of the following form

$$E_i - Q - A_i^T E_i A_i + A_i^T E_i B_i (R + B_i^T E_i B_i)^{-1} B_i^T E_i A_i = 0. \tag{33}$$

Using defuzzyfication method [18] we obtain the following matrices.

$$A = \begin{pmatrix} 0.995 & 0.005 & 0.0028 & 0 \\ 0.005 & 0.395 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \; B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \end{pmatrix} \text{ and } C = \begin{pmatrix} 0 \\ 0.2437 \\ 0 \\ 0 \end{pmatrix}$$

Using MATLAB built-in function dare to solve (33) and defuzzyfication method we obtain

$$K = \begin{pmatrix} 0.0104 & 0.0013 & 3.3354 & 0 \\ 0 & 0 & 0 & 2.2468 \\ 0.1880 & 0.0125 & 0.0110 & 0 \\ 0.0296 & 0.0401 & 0.0031 & 0 \end{pmatrix}$$

The variation of controls of cardiovascular-respiratory system are represented in the Figure 4 while the Figures 5 and 6 present response of physical activity due tothese controls.

According to numerical results, the controls of cardiovascular-respiratory system, heart rate and the alveolar ventilation increase since the onset of physical activity until they reach a stabilized state (See Figure 4). This behaviour of these control dur- ing physical activity can help to avoid or even heal non severe hypoxemic-hypoxia. Furthermore, on the one hand ventilation increases and gas is supplied and regu- lated through the body but on the other hand increase in heart rate and ventilation rate result in an adequate and regular supply of both oxygen and carbon dioxide in exercise. The Figure 5 shows that tissue and venous dioxide carbon concentration decrease. This is due to brut increase of ventilation at the beginning of the physi- cal activity which is followed by a gradual increase of ventilation. Accumulation of lactic acid may allow cardiovascular-respiratory system to become less ventilated.

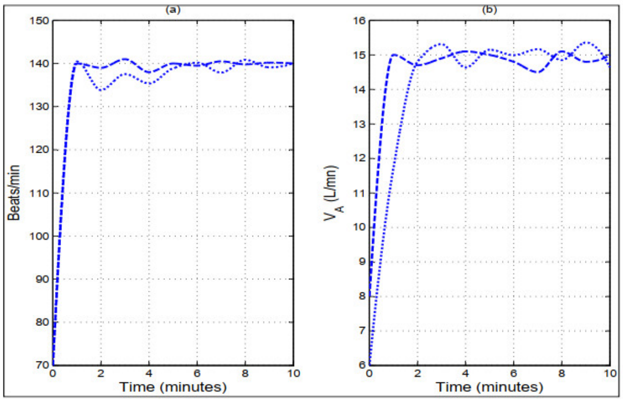

Figure 4. Variation of heart rate (a) and ventilation rate (b) for a 30 years old woman performing jogging. The curves in dotted line represent the parameter for the direct approach. The curve dashed line show the parameter for the approach integrating the fuzzy logic strategy

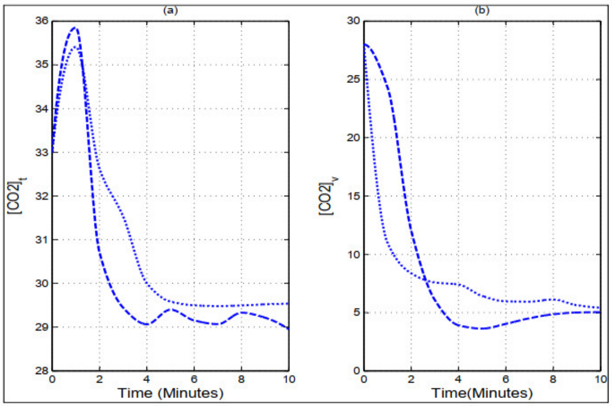

Figure 5. Variation of carbon dioxide in tissue (a) and in vascular (b) for a 30 years old woman performing jogging. The curves in dotted line represent parameter obtained using direct approach. Approach integrating fuzzy logic strategy allows to get curve of parameter in dashed line

This physiological phenomenon causes the increase (respectively decrease) of $PaCO_2$ (respectively PaO_2). This doesn't happen for a 30 years old woman performing jog- ging as shown in the Figure 6. Furthermore, before being stabilized at equilibrium value, arterial partial pressure of dioxide carbon (respectively oxygen) decreases (increases).

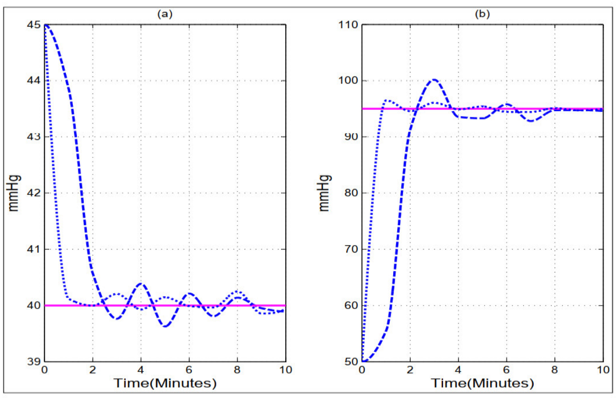

Figure 6. Variation of optimal arterial partial pressure of carbon dioxide (a) and oxygen (b) for a 30 years old woman performing jogging. The curve in solid line represents equilibrium value of parameter of cardiovascular-respiratory system. The curve in dotted line indicates optimal parameter obtained using approach integratingthe fuzzy logic strategy while direct approach allows to get the curve in dashed line that illustrates optimal parameter of cardiovascular-respiratory system

CONCLUDING REMARKS

The results obtained in this work are rather satisfactory. To test efficiency of ap- proach integrating fuzzy logic strategy, the numerical results are compared to ones obtained using direct method. The numerical finding show that those two used methods are satisfactory and closed since they confirm empirical evidence. In par- ticular, the response of controls of cardiovascular-respiratory system to exercise can be modeled and a feedback is approximated by the solution of a linear quadratic problem. Consequently, physical activity reduces the risk of hypoxemic-hypoxia or any cardiovascular-respiratory disease. Furthermore, physical activity induces im- portant changes in the stabilization of cardiac, vascular and blood tissue. Approach integrating the fuzzy logic strategy is very important for solving optimal control problem. In particular, it gives optimal trajectories of cardiovascular-respiratory system in the same way it ensures performance of this system.

REFERENCES

1. Samuel, Jacob; Franklin, Cory (2008). Common Surgical Diseases. New Yotk: Springer. pp. 391–94. doi:10.1007/978-0-387-75246-4 97. ISBN 9780387752457
2. West, John B. (2008). Respiratory Physiology: The Essentials (8th ed.). La Jolla: Wolters Kluwer Lippincott Williams & Wilkins. pp. 88–89.
3. Hiraga T. Hypoxic Microenvironment and Metastatic Bone Disease. Int J Mol Sci. 2018 Nov 9;19(11). pii: E3523. doi: 10.3390/ijms19113523.
4. Watts ER, Walmsley SR. Inflammation and Hypoxia: HIF and PHD Isoform Selectivity. Trends Mol Med. 2018 Nov 12; pii: S1471-4914(18)30202-8. doi: 10.1016/j.molmed.2018.10.006
5. Keuski BM. Updates in diving medicine: evidence published in 2017-2018. Un- dersea Hyperb Med. 2018 Sep-Oct;45:511-520.
6. Lahiri S., A.Mokashi, R.G.Delaney and A.P.Fishman, Arterial PO2 and PCO2 stimulus threshold for carotid chemoreceptors and breathing, Respi- ration Physiology, Volume 34, Issue 3, September 1978, Pages 359-375, https://doi.org/10.1016/0034-5687(78)90134-2
7. Bhattacharyya N.K., D.J.C.Cunningham, R.C.Goode, M.G.Howson and B.B.Lloyd, Hypoxia, ventilation, PCO2 and exercise, Respiration Physiol- ogy,Volume 9, Issue 3, June 1970, Pages 329-347.
8. Dempsey J.A., and Wagner P.D., (1999), Exercise-induced arterial hypoxemia. J. Appl. Physiol. 87:1997–2006
9. Amann M., and Calbet J.A, (2008), Convective oxygen transport and fatigue, J. Appl. Physiol. 104:861–870
10. Zadeh L.A., Fuzzy sets, Information and Control, Volume 8, Issue 3, June 1965, Pages 338-353
11. Simoes, M. G., (2010), Introduction to fuzzy control, Colorado School of Mines, Engineering Division, Golden, Colorado, 8, 18-22.
12. Gutierrez G., A Mathematical Model of Tissue–Blood Carbon Dioxide Exchange during Hypoxia, Am J Respir Crit Care Med Vol 169. pp 525–533, 2004.
13. Timischl S., J. J. Batzel, and F. Kappel, Modeling the human cardiovascular- respiratory control system: an optimal control application to the transition to non-REM sleep, Spezialforschungsbereich F-003 Technical Report 190, Karl- Franzens-Universitat, 2000.

14. Douglas A.R., N.L Jones, J.W. Reed, Calculation of whole blood CO2 content, J Appl Physiol 1988;65:473–477.
15. Severinghaus J.W., Simple, accurate equations for human bloodO2 dissociation computations, J Appl Physiol 1979;46:599–602.
16. Giovannini I., C. Chiarla, G. Boldrini, M. Castagneto, Calculation of venoar- terial CO2 concentration difference, J Appl Physiol 1993;74:959–964.
17. Sugeno M. et K. Murakami, Fuzzy parking control of model car, in 23rd IEEE Conf. on Decision and Control, 1984.
18. Takagi T. and M. Sugeno, Fuzzy Identification of Systems and its Applications to modeling and Control, IEEE Trans. Systems Man And Cybernetics, vol. 15, pp. 116-132, 1985.
19. Yakowitz S. J., The Stagewise Kuhn-Tucker Condition and Differential Dy- namic Programming, IEEE Trans, vol.31, N◦1, pp. 25-30, 1986
20. Park M. I., E. Kim, S .Ji and M. Park, A new approch to fuzzy modeling, IEEE Trans. Fuzzy Syst, vol. 5, no. 3, pp. 328-337, August 1987.
21. Jacobson D., D. Lele, J. L. Speyer, New necessary conditions of optimality for control problems with state-variable inequality constraints, Journal of Mathe- matical Analysis and Applications, Vol. 35, pp. 255-284, 1971.
22. Ntaganda J. M. and B. Mampassi, Modelling blood partial Pressures of the hu- man cardiovascular respiratory system, Applied Mathematics and computation, Vol.187, pp 1100-1108, 2007.
23. Krichen Masmoudi N. and N. Derbel, Optimal control of nonlinear systems by fuzzy logic, ISCIII, Nabeul, Tunisia, 2003.
24. Kwakernaak,. R. Savan, Linear. Optimal. Control. Systems,. Wiley. Inter- Science,. New. York,. 1972.

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