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Research Article

MODELING FOR ORTHO-PLANAR FLEXURE BASED COMPLIANT MECHANISM

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Abstract

This paper presents the design considerations of ortho-planar flexure hinge that has the vertical motions of platform relative to the base boundary with no rotation. The design concept is its non-rotating vertical motion, eliminates the problem of rotation against adjoining surfaces and is less sensitive to variation in the state of assemblies. The design equations are presented to identify different configurations and the force-defection relationships. The objective of this work was to apply the mathematical method to design an ortho-planar flexure hinge while the design should have minimum mass and at the same time satisfy a set of constrained displacement. The mathematical and topological processes showthe layout design under small displacement conditions, the output displacement, maximum stress magnitude, and the maximum stress of linear elastic assumption. However, the mass fraction and the layout as the result of the optimization process may be different. As for larger displacement, the maximum stress of linear elastic material appeared some times higher than the maximum stress of the small displacement model. Thus, the design consideration of topology optimization scheme may be selected by the linear or nonlinear material and deformation models.

Keywords: Ortho-planar flexure, Flexure hinge, Compliant mechanism, Topology optimization.

INTRODUCTION

A flexure is a monolithic compliant element that connects two or more assumed rigid links, allowing for selectively chosen movements. Flexures are engineered to be compliant for specific relative rigid link movements, the mechanism Degrees of Freedom, while stiff in other mechanism degrees, the mechanism Degrees of Constraint. A flexure hinge consists of a flexible, slender region between two adjacent rigid parts that undergo relative limited rotation in a mechanism and is the important constituent of lumped compliant mechanisms. Through elastic deformation, flexures offer extreme position repeatability within a limited range of motion in their degrees of freedom, while constraining motion in the degrees of constraint. Topology optimization proves a prospective tool for the design of short-stroke flexures, providing maximum design freedom and allowing for application-specific requirements. Flexure hinges have several advantages over conventional rotational joints due to being monolithic with the rest of the mechanism.[1,2] They have no friction losses, no need for lubrication, and no backlash; they do have compactness. Therefore, flexure hinges are widely used in translation micro-positioning stages, scanning tunneling microscopes, high-precision cameras, robotic micro-displacement mechanisms, and especially in micro-electromechanical systems (MEMSs).[3–5] The primary design requirement of a short-stroke flexure is the relative stiffness between the mechanism Degrees of Freedoms and Degrees of Constraints. Secondary considerations are range of motion, axis drift, deformation and stress, fatigue, volume and mass, as well as the sensitivity of those aspects to, e.g., manufacturing errors. The synthesis methods often used for rigid body mechanisms, cannot straightforwardly be applied to compliant mechanisms. There is always mechanical stress involved in any motion, and the behavior is dependent on the loading condition.

This implies that kinematics (motion) and kinetics (load case) must be treated simultaneously. As a result, the concept of mechanism DOFs fades in compliant mechanisms, because they behave differently for any loading conditions. Systematic flexure synthesis methods rely on kinematic approaches, such as rigid-body replacement techniques or the freedom and constraint topology method. As the most important components of compliant mechanisms, various cross section profiles of flexure hinges have been studied. The aforementioned design of flexure hinges is based on a given shape or a known topology. The performances of flexure hinges are mainly determined by the shape of cross section. Designers' experiences have a large impact on the performances of hinges. A general empirical equation for evaluating the stress level is also developed based on the finite element method. The analytical model predictions are confirmed by FEA results and experimental measurement data within 5% uncertainty. Numerical descriptions demonstrate the versatility in flexure types and the extend ability of additional design requirements.

Topology optimization of compliant mechanism

Topology optimization is a computational design methodology which iteratively utilizes finite element analysis to generate the optimal distribution of material within a fixed design domain that minimizes a prescribed objective function and satisfies a series of constraint functions. Here, the solid isotropic material with penalization(SIMP) method allows each finite element to occupy a continuous density space between 0 and 1 in the design density of flexure hinges due to its high computational efficiency. The finite element method is used to discretize the topology optimization problem. In the standard displacement-based linear FEA, the displacement field $\boldsymbol{u}$in the design domain is approximated by nodal interpolation such that

$$u = N(x)u_e \qquad (1)$$

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where $N(x)$ is a matrix of global shape functions and u_e is the corresponding element nodal displacement vector. Using geometric equations, the strain-displacement relationship is derived as

$$\varepsilon = Bu_e \quad (2)$$

where ε is the element strain vector and B is the standard B-matrix constructed from the global shape functions. According to Hooke's law, the element stress vector is written as follows:

$$\sigma = E_e\varepsilon = E_e B u_e \quad (3\text{-}1)$$

When applied to structural problems, the SIMP method defines a heuristic relation between an element's elastic modulusand its density, which forms the design variable.

$$E_e(x_e) = E_{void} + \rho_e^p (E_{solid} - E_{void}) \quad (3\text{-}2)$$

Where ρ_e is the density of element e, and $\rho_e \in [0, 1]$, $x_e = 0$ and $\rho_e = 1$ produce void and solid element of moduli E_{void} and E_{solid}, respectively. A small, non-zero E_{void} is employed to avoid singularities. Further, p is the penalty exponent, where for $p \geq 3$ the design approaches a binary solution

The optimization problem is solved by iteratively evaluating the finite element equilibrium equation:

$$k_e u_e = F_e \quad (4)$$

where $k_e = \int_\Omega B^T E B d\Omega$ is the element stiffness matrix and F_e is the nodal force vector. As the hinge displacements in this study are typically small, a linear solver is used to produce a tractable solution. In hinges with large displacements, geometric non-linearities may reduce the accuracy of results.

Generally, the work done by external forces is defined as mean compliance. Its mathematical formulation can be expressed as,

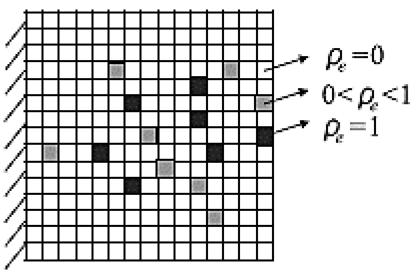


Fig. 1. Discrete design domain of finite element

$$C = \int_V f^B u dV + \int_S f^S u dS + \sum_i F_i u_i \quad (5)$$

where f^B is the body force, and f^S is the surface traction. F_i and u_i are point force and displacement of the i-th degree of freedom, respectively.

In the discrete model of the SIMP approach, the design domain is discretized by N four-node square finite elements (see Fig. 1(b)) and the material density ρ_e in each element is a design variable. Each node has two degrees of freedom which are translations in the x and y directions. From Fig. 1(a), it is noted that the material density distribution ranges from solid (black) $\rho_e = 1$ to void (white) $\rho_e = 0$ in every element. In addition, a penalty factor penalizes intermediate (grey) elements with $0 < \rho_e < 1$ driving the structure towards a white-and-black configuration. In this formulation, the material properties (such as elastic tensor, stiffness matrix, and element densities) are connected. A relation between element density ρ_e and elastic tensor E_{ijkl} is expressed by $E_{ijkl} = \rho_e^p E_{ijkl}^0$, where E_{ijkl}^0 is the elastic tensor of the solid material and p is the penalty factor.

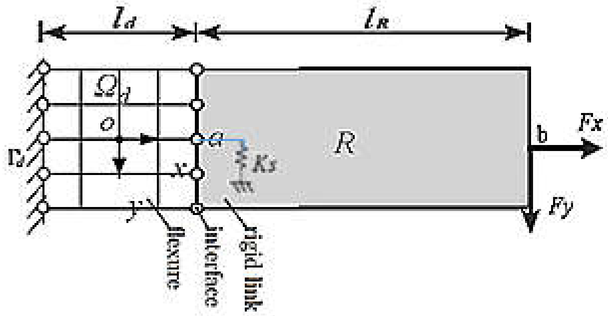


Fig. 2. A sketch of the design domain and force loading condition

The design domain of the flexure hinge is shown in Fig. 2, where Ω_d is the design domain which is set to be a square, and R is the rigid domain (design variables are set to be 1) which belongs to the non-design domain. The design domain is fixed at boundary Γ_d and is symmetric with respect to the x-axis. Loads F_x and F_y are acted at input port b. A 2D design space is used to enable a relatively large planar space to be solved, and because the resulting structures are expected to be primarily planar. These mechanism degrees define prescribed nodal displacements at the interfaces between rigid link and flexure (e.g. a unit displacement in y-direction between top and bottom interfaces for mechanism degree). The assumption that these interfaces are rigid is valid if the links can be considered much stiffer compared to the flexure. As such, the mechanism degrees correspond to the relative rigid body motions of the interfaces. Since the deformations of a flexure hinge are small, a generic hinge can be modeled and analyzed as a small-deformation fixed-free Euler-Bernoulli beam subjected to axial and bending effects produced by forces and moments. In general, the compliance of point a is used to indicate the compliance of a flexure hinge, which can be calculated by the displacement of point a and the loads applied at this point. However, in the implementation of flexure hinge topology optimization, the forces F_x and F_y are applied at point b of the rigid link instead of at point a. Thus, the forces and moment applied at point a are equivalent to the form of the forces F_x and F_y. The space is parametrized by an FEA mesh. Square elements of equal side-length are selected for their ability to efficiently span a square design space. For flexure hinges, when the stiffness of flexure hinges in the x-direction is as large as possible and the stiffness in the y-direction is as small as possible, the hinges are closer to the ideal joints. Therefore, we need to maximize the compliance in the y-direction and minimize the compliance in the x-direction. Since only point forces are considered in topology optimization, the compliances Cx and Cy can be calculated from Eq. (5) as follows:

$$C_x = F_x^T u_x,\ C_y = F_y^T u_y \quad (6)$$

where Fx and F_y are the external force vectors, and u_x (produced by Fx) and u_y (produced by F_y) are the displacement vectors for the mechanism in its equilibrium position. The objective function of optimization problem can be stated as follows:

$$\text{minimize: } \frac{c_x}{c_x^0} - \frac{c_y}{c_y^0} \tag{7}$$

where C_x^0 and C_y^0 are the initial compliances in topology optimization.

The optimization is performed using the method of moving asymptotes, a convex approximation method developed for efficiently solving structural optimization problems. A constraint function of the optimization problem for flexure hinges can be written as follows:

$$\varphi = \frac{1}{2}\left(\frac{u_b^y}{u_a^y} - \frac{l_d+2l_R}{l_d}\right)^2 \le \varphi^* \tag{8}$$

where u_a^y and u_b^y are the displacements of points a and b along y-direction due to the force F_y, respectively. is a given small positive constant.φ^*The finite element discrete form of the topology optimization problem for flexure hinges can be formulated as follows:

$$\text{Find: } \rho = [\rho_1, \rho_2, \ldots\ldots, \rho_N]^T \in R^n \tag{9}$$
$$\text{Minimize: } \Psi(\rho) = C_x - C_y$$

subject to $Ku_x = F_x$,
$Ku_y = F_y$,
$\varphi \le \varphi^*$,
$\sum_{e=1}^{N} \rho_e v_e \le V^*$, $0 < \rho_{min} \le \rho_e \le 1$, $e = 1,2,\ldots,N$

where V^* denotes the allowed volume fraction and K is the global stiffness matrix which can be written as

$$K = \sum_{e=1}^{N}\left(E_{void} + \rho_e^p\left(E_{solid} - E_{void}\right)\right)k_e + K_S \tag{10}$$

Where k_e is the element stiffness matrix for unit material stiffness,Ks is the stiffness of the output flexure hinge (as shown in Fig. 2) in global level. The artificial flexure hinge modelis often used at the input and output ports to model the clearance between the workpiece and the mechanism and also simulate the reaction force from workpiece. Figure 3 shows the final topology results of flexure hinges with different Ks. For clearly plotting the results of topology optimization, only part of the rigid domain R is plotted.

The term $\partial u/\partial \rho_e$ can be determined from $\boldsymbol{Ku} = \boldsymbol{F}$ with K being the global stiffness matrix and $\boldsymbol{F}$ being the vector of external forces, by differentiation with regard to the design variable,

$$\frac{\partial K}{\partial \rho_e}\boldsymbol{u} + \boldsymbol{K}\frac{\partial u}{\partial \rho_e} = \frac{\partial F}{\partial \rho_e} \tag{11}$$

$$\frac{\partial u}{\partial \rho_e} = K^{-1}\left(\frac{\partial F}{\partial \rho_e} - \frac{\partial K}{\partial \rho_e}\boldsymbol{u}\right) \tag{12}$$

where $\frac{\partial K}{\partial \rho_e}$ sums up the derivatives of the element stiffness matrices $\mathbf{k}_e$.

Theoretical considerations for compliant mechanism

Ortho-planar compliant flexure can have different configurations according to the number of flexible segments (Fig. 3). The minimum possible number of segments is two, in order to align the structure in parallel above the compliant flexure with the structure below the compliant flexure. The design of structure of the ortho-planar compliant flexure should be flexible enough to achieve displacement, and, at the same time, robust in order to avoid breaking. It is known [6,7] that four segments do not provide better stability of the structure compared to three legs. Constructing an ortho-planar compliant flexure from a higher number of flexible segments is possible, but with a keen interest to achieve a miniaturized sensing structure, we chose a three-segmented approach.

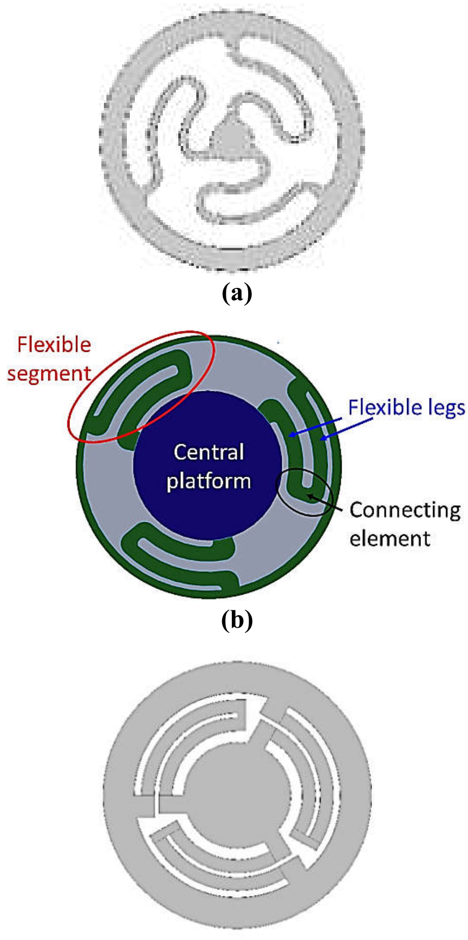


Fig. 3. Design space of ortho-planar flexure with different Ks. (a) Ks= 0.01, (b) Ks= 0.03, and (c) Ks= 0.06.

Usually, the circle, ellipse, hyperbola, or other common shapes are selected as the profiles of flexure hinges, but they may not be the best topologies of flexure hinges. From Fig. 3, it can be seen that nearly the same topologies are obtained and the shapes of these topologies are rather different from the shapes of the conventional hinges. The profile of the flexure hinge is formed by three different segments (as shown in Fig. 4). Note that the middle segment is slightly different with different flexure hinge stiffness Ks, so we can obtain various flexure hinges with different performances by changing the thickness and width of the middle segment. Therefore, a new type of flexure hinge is designed based on topology optimization results. The theoretical analysis is formulated based on several simplifying assumptions that are summarized in the following:

- The flexure hinges comprise three portions: the left and right segments are polynomial curves and the middle segment is of constant cross section.
- The flexure hinges are symmetric with respect to the longitudinal axis and asymmetric with respect to the middle transverse axis.
- The flexure hinges are designed to be applied in two dimensional compliant mechanisms. They are formulated to characterize in-plane motions and only three degrees of freedom are considered: two translations in the x- and y-directions, and one rotation around the z-axis.
- Since all deformations of flexure hinges are small, the hinges are modeled and analyzed as small-displacement fixed-free Euler-Bernoulli beams subjected to bending and axial effects produced by forces, moments, and axial loads. Shearing and torsional effects are not taken into account.
- The boundary conditions for flexure hinges are fixed-free, the left end of flexure hinges is fixed, and the other end is free.

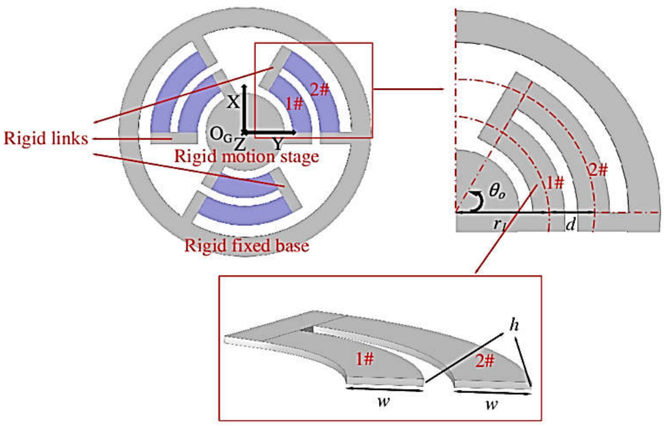
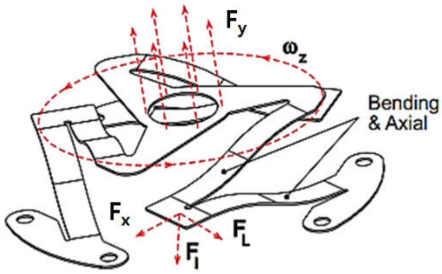
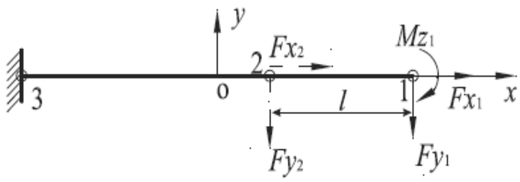


Fig. 4. Detailed geometric information of curved-beam-based Ortho-planar flexure hinge

Compliance is one of the main parameters for flexure hinge design. The compliance equations of flexure hinges are derived by applying the basic theory of mechanics of materials. For a combination of loads, the principle of superposition can be used to calculate the total deformation since the deformations in all directions are small enough. Based on the previous assumptions, the hinge can be modeled as a cantilever beam. Schematic representation of a flexure hinge with loading is shown in Fig. 5. We can evaluate the parameters of the ortho-planar flexure hinge elements to show how they influence the spatial deflection of the overall structure. The deflection of a single flexible leg can be expressed again from the rectangular beam equation. Point 1 is the free end with loads, point 2 is the center of rotation, and point 3 is the fixed end; they correspond to positions 1, 2, and 3 in Fig. 5, respectively. Torsional moment shown in FIG. 6-(a), the eccentric nature of the leg design turns the intermediate platform into a moment arm which creates torsion along the leg segments. These stresses may be more prevalent in the pure bending type than in other leg designs. Torsion may also be present in the intermediate platforms. Torsional behavior in the leg segments will increase as the moment arm(e shown in Fig. 6-(a)) increases. Torsion in the intermediate platform will increase with the orthogonal load. Shear stress will be present near the transition geometry due to a tearing phenomena created by orthogonal loads. See Fig. 6-(b). This type of stress will produce a stress concentration found at the transition radii. This shear area in the intermediate platform has the highest stress while subject to only orthogonal loads. An ortho-planar flexure hinge is a kind of compliant mechanisms that utilizes the out-of-plane deformation of its flexible limbs. The ortho-planar flexure hinge has many advantages, including capability of being fabricated from a single piece of material that reduces manufacturing costs with its compact form, enabling it to be used in a highly confined space.

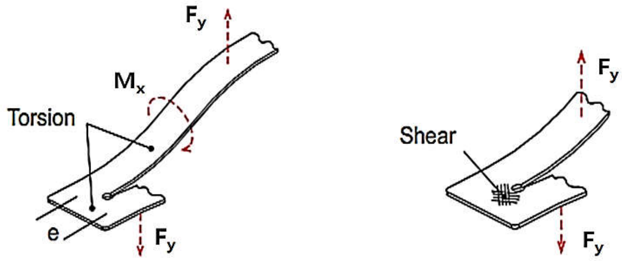


(a)



(b)

Fig. 5. Schematic representation of a flexure hinge with loads. (a) Bending and axial loads ortho-planar compliant mechanism, (b) Loading condition.



(a) (b)

Fig. 6 (a) Torsional stresses in leg segments and intermediate platform from eccentricity of pure bending leg design (b) shear stresses in intermediate platform.

By defining the load vector as,

$$F = [M_{z1}, M_{x1}, F_{y1}, F_{x1}]^T \tag{13}$$

and the corresponding displacement vector as

$$X = [\theta_1, \varphi_1, y_1, x_1]^T \tag{14}$$

the following relationship of displacement-load at the free end is obtained:

$$X = CF \tag{15}$$

where C is the compliance matrix of the flexure hinge, which can be expressed as follows:

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 \\ C_{21} & C_{22} & C_{23} & 0 \\ C_{31} & C_{32} & C_{33} & 0 \\ 0 & 0 & 0 & C_{44} \end{bmatrix} \tag{16}$$

where $C_{ij} = C_{ji}$, according to the reciprocity principle. Each element in the compliance matrix is called a compliance factor. Based on Castigliano's second theorem, the displacement load relationship can be formulated as follows:

$$\begin{cases} \theta_1 = \dfrac{\partial U_\varepsilon}{\partial M_{z1}} \\ \varphi_1 = \dfrac{\partial U_\varepsilon}{\partial M_{x1}} \\ y_1 = \dfrac{\partial U_\varepsilon}{\partial F_{y1}} \\ x_1 = \dfrac{\partial U_\varepsilon}{\partial F_{x1}} \end{cases} \tag{17}$$

Since the flexure hinge is subjected to bending and axial load, the elastic strain energy U_ε comprises bending and axial terms and can be expressed as,

$$U_\varepsilon = \frac{1}{2}\int_0^l \left\{ \frac{\left[M_{z1} + (l(x) - x)F_{y1}\right]^2}{EI(x)} + \frac{M_{x1}\, l(x)}{GJ(x)} + \frac{F_{x1}^2}{EA(x)} \right\} dx \tag{18}$$

where $I(x)$, $J(x)$ and $A(x)$ are the area moment of inertia, polar moment of inertia and area with variable shape functions of each flexure bridge. $l(x)$ is the length of the flexure hinges. Each flexible flexure hinge element is made up of a rectangular beam with cross-sectional parameters—width w and height h. the path of x-direction changes along the flexure bridges.

Conclusion

Ortho-planar mechanisms are comprised of both compliant and rigid-body mechanisms. Ortho-planar mechanisms are defined as mechanisms in which all the links can be located simultaneously in a single plane. Compliant ortho-planar flexure hinges have been used to provide force-displacement behavior in compact spaces. Through the design consideration of mathematical and topological approaches, compliant ortho-planar flexure hinges can potentially eliminate many of the problems associated with traditional ortho-planar flexure hinges. Some of these advantages include less rotation of the platform, no required clearance, increased displacement, and improved fatigue life. This paper has been to investigate the behavior of compliant ortho-planar flexure hinges subjected to complex loads, specifically their behavior for lateral stability, the effects of stress stiffening, and the effects of inertial loading, and may show the qualitative look at the design space and imposed limitations, parameter influence (flexure hinge geometry), loads, stresses and the applicability of current design tools.

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