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Research Article

MATHEMATICAL MODELING OF COVID-19: A BRIEF SURVEY OF EARLY MODELS

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Abstract

The SARS-Cov-2 caused Covid-19 has been the major health concern of the entire world for more than 2 years now. Various pharmaceutical and non-pharmaceutical strategies are still being tried out and employed to bring an end to the spread of this deadly disease. Mathematical modeling techniques also have been widely used to study the dynamics and progression of the pandemic and to propose control measures. Thousands of modeling works, employing various techniques, were developed even in the early months after the emergence of the disease. The objective of this paper is to provide a brief survey of some of the early papers in this area, detailing the country wise distribution, the mathematical methods used and the proposed recommendations for disease control. In particular, methods based on integer and fractional order differential equations, statistical and stochastic techniques are delineated.

Keywords: Covid model, ODE models, Nonlinear models, fractional equation models.

INTRODUCTION

Corona virus disease, abbreviated as COVID, has shaken the whole world beyond unimaginable ways. Identified first among a few pneumonia cases in December 2019, the disease has rapidly spread to the entire globe forcing almost all countries to declare emergencies, lock downs, curfews, social distancing, use of face masks, and the sort. Corona viruses are not new to the world, but a new corona virus SARS-Cov-2 has been the agent for COVID-19. It is an undeniable fact that the present generation has undergone an unprecedented array of experiences for more than 2 years after the first cases of Covid were reported. The response of the entire world to arrest the spread of this pandemic was quick and timely. Along with scientists and health and medical professionals, mathematical modelers also have made significant contributions in the disease control. Mathematical modeling of infectious diseases has been an active field of applied mathematical and statistical techniques for a couple of centuries now. This field saw an exponential growth of researches and publications in the 20th century especially after the emergence of the devastating HIV/AIDS among the humans in early 80s. As the second decade of the 21st century witnesses the outbreak of another global health threat, enormous volume of mathematical modeling studies have been undertaken to address the situation based on various modeling techniques. One is astonished at the enormous number of publications in this area even during the early months of the pandemic. This paper aims at a brief survey of some of the early mathematical models for studying the dynamics, spread and control of Covid 19.

METHODOLOGY

The methodology followed in this survey is as follows: A search on the ScienceDirect website on 16th April 2021 was conducted using the keywords `COVID mathematical model' and research articles were identified. The search elicited more than 2300 entries which shows the enormous modeling studies that have been undertaken in this area in those early months of Covid-19 pandemic. The articles were further filtered to obtain those published in 2021 which gave 1829 results. These were sorted by relevance and the most relevant 50 were chosen for this current study. The remainder of the paper is organized as follows:

In the next section we classify the selected 50 papers based on the country/region wise data used for simulations specifying the associated time period of the data. In the following section we will give an exposition of the mathematical methods and tools used in the selected models. The findings and recommendations for Covid control proposed by these modeling studies are described in the subsequent section, which will be followed by a concluding section.

Country wise classification of the data sources

The pan global nature of the COVID-19 epidemic is also reflected in the data sources being used for the modeling studies. A good number of modeling studies rely on data from China especially from Wuhan, the epicenter of the disease. Ameen *et al.* [1] used data reported by the National Health Commission (NHC) of China in a comparative manner for three periods during January to November 2020. Nisar *et al.* [2] took real data, reported by WHO and worldometer records, against confirmed infected and death cases per day for the initial 67 days in Wuhan city from early January to early March 2020. The modeling which studied COVID-19 epidemiology, clinical manifestations, and prevention and treatment effects by Xiao *et al.* [3] was based on the clinical data from 500 COVID-19 patients admitted to a hospital in Sichuan during a three month period beginning from February 2020. Panwar *et al.* [4], in their COVID modeling using non-singular fractional derivatives, used Wuhan city as an example for numerical comparisons. Abdullah *et al.* [5] also used data from Wuhan for 68 days from early January to early March 2020 to compare their simulation results. The real data of disease outbreak of Wuhan city for 66 days during the same period as above was used by Ali *et al.* [6] for their qualitative

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analysis of fractional order model. A model, communicated in a correspondence by Hou *et al.* [7], which described-spatiotemporal transmission dynamics of COVID-19 at the early stage in China used the epidemiological data collected from the National, Hubei Provincial and Wuhan Municipal Health and Family Planning Commissions and the population movement data estimated by Baidu Huiyan APP. Lu Bai *et al.* [8] evaluated the trend of the COVID-19 epidemic for various scenarios of work resumption strategies for the returning residents in Shenzhen, using data from Shenzhen Municipal Government.

Many COVID modeling studies have used data from India as well. Kumar *et al.* [9] employed a statistical approach with data for first 115 days of spread of Covid-19 in India from April 01, 2020 onwards, on the basis of information available through authenticated sources, for projecting 20 days forward. The data for the period March-August 2020 from the states of Uttar Pradesh, Delhi, Bihar, Karnataka, Maharashtra, based on AarogyaSetu app, was availed by Sharma *et al.* [10] in their fuzzy logic model. Askar *et al.* [11] depended on data from Ministry of Health to explain, understand and forecast COVID-19 in India using a fractional-compartmental model. Foy *et al.* [12] who studied vaccine allocation strategies in India developed their findings based on confirmed incident cases of COVID-19 from 10 most populous states during July-August 2020. A fractal-fractional modeling approach [13] predicted future trends in the behavior of the epidemic of detected cases and deaths in India. Bhadauria *et al.* [14] investigated the impact of lockdown on COVID dynamics in India, starting from 24 March 2020, by taking some of the parameter values from previous literature and the rest assumed. A few studies have been identified elsewhere from some other South Asian countries also. Peter *et al.* [15] used COVID incidence data from Pakistan for 45 days during July-August 2020, taken from Government sources, whereas Din *et al.* [16] relied on worldometer records for Pakistan during the period February to September 2020 in order to simulate the compartmental model with convex incidence rate. The fractional order model by Ali *et al.* [17] also made use of real data from Pakistan, mainly from WHO records, to establish their results. The key parameters related to COVID-19 transmission and control in Nepal, which shares an open border with India, were estimated with the help of data from ministry of health and population in Nepal by Adhikari et al. [18]. Shahrear *et al.* [19] modeled the outbreak of COVID in Bangladesh with data taken from worldometer records for the period March to October 2020.

COVID models were developed for African countries also by researchers. Musa *et al.* [20], who modeled the effect of awareness programmes for COVID control, relied on data from Nigeria for the period March to June 2020 from available sources, whereas the same country data were used by Baba *et al.* [21] in their effort to assess the imposition of lock down on the dynamics of COVID in Nigeria. Ahmed *et al.* [22] also used the data provided in the Nigerian center for disease control and WHO in order to validate their compartmental model. Real data from Ethiopia for the period March to June 2020, collected from WHO records, were used by Gebremeskel *et al.* [23]. COVID dynamics in Ghana were modeled by Dwomoh *et al.* [24] with the help of data obtained from Ghana Health Service. Djomegni *et al.* [25] made a casestudy using data from South African scenario. The stochastic model proposed by Danane *et al.* [26] used data given by the health ministry of Morocco to estimate the relevant parameters. Epidemiological data from Morocco as well as from three other countries, namely Belgium, Netherlands and Russia, were used by Lmater *et al.* [27] to validate the proposed SIDR model. There are many other countries across Europe, Americas and Asia also taking data from where COVID-19 modeling studies have been undertaken. Spain [28-29], Portugal [30-31], Egypt [32-33], Turkey [34], United States [35-37], UK [38], Brazil [39], Korea [37], Japan, Iran [40], Canada [41], and Serbia [51] are a few among them. Apart from all the above country specific data simulations, many modelers have availed global data for the calibration and verification of their data. See for example [42-43]. As we have seen above, almost all the cited works have used data for the period March- December 2020 for the studies, which can be considered initial data for the Covid 19 pandemic.

Mathematical methods used

In this section, we briefly outline the mathematical methods and techniques used in the selected COVID-19 publications under study. Most of the studies employed compartmental models using integer order and fractional order systems of differential equations (DEs). The basic approach followed is the SIR approach (Susceptible-Infected-Recovered) with individual studies adopting relevant variations and modifications.

Integer order differential equations

A sizeable number of compartment models made use of integer order differential equations. Adhikari *et al.* [18] used an SQEIR model where E refers to the Exposed class and Q refers to the Quarantined class. Moreover, the infectious class I of people were divided into two: recorded infectious and non-recorded infectious. Using this model the authors evaluated the control strategies implemented by the government of Nepal, namely, (i) Border screening and quarantine, (ii) Lockdown, and (iii) Detection and isolation. Peter *et al.* [15] also availed a SEIQR traditional model investigating specially the invariant region, equilibrium state, basic reproduction number, global stability of the free disease equilibrium and performing sensitivity analysis of the relevant parameters. A modified SEIQR model with special classes for symptomatically infectious and asymptomatically infectious is constructed by Olivares *et al.* [28] for uncertainty quantification of COVID dynamics with mass vaccination strategy, who also employ a statistical moment based polynomial chaos expansion for sensitivity analysis. Musa *et al.* [20] in their effort to assess the effect of public awareness on the dynamics of COVID-19 infection, used an SEIHR model where H refers to the hospitalized population. This model further classified susceptibles into aware and unaware, infected into asymptomatically and symptomatically infectious, and hospitalized into mild and severe, thus having a total of 8 compartments. There was also a class for deceased humans. The model created for forecasting COVID situation in Ethiopia [23] used a simple SIRD approach where D referred to the dead individuals. The Sobol's method of sensitivity analysis on input quantities of interest to study their impact on the output quantities of interest for an SEIRD model was the characteristic of the work by Zhang *et al.* [44]. Baba *et al.* [21] in their SIL model divided both the susceptible and the infected populations into those under lock down and those not under lock down and denoted the cumulative density of the lock down programme by L(t). Ahmad *et al.* [22] adopted a novel approach by including a resistive (healthy) class H along with the quarantined class Q for their SHIQ model. Din *et al.* [16] opted for an SIR model but with the convex incidence rate and the Non Standard Finite Difference Scheme for simulations. In their study for comparing the vaccination strategies in India, Foy *et al.* [12] considered an age specific SVEIAQRD modeling approach stratifying the classes based on various age bands of 10 year difference, including also classes for vaccinated (V), asymptomatic (A), and symptomatic (I) along with exposed (E), quarantined (Q) and died (D). Similar to the above models, an SEIQHRS approach has been adopted also by Dwomoh *et al.* [24] who studied governmental level and individual level interaction impact in Ghana. Having a special class of protected susceptibles, who are risk free individuals to be infected, along with the usual other classes has been the peculiarity of the model by Djomegni *et al.* [25] for South Africa. Bai *et al.* [8] in their study of work resumption strategies for Shenzhen depended on a SEITRD model combined with population mobility. The study on the combined effect of lockdown, contact tracing and quarantine on the COVID-19 dynamics in India however used a simple SIQ model [14]. COVID analysis and prediction in Bangaldesh was undertaken using a SEQIRP model [19] where P here refers to the people who died of the disease. Lmater *et al.* [27] model the pandemic for a machine learning forecasting with relaxation scenarios of countermeasures using SIDR model where D refers to the diagnosed class. A case study model of Brazil [39] for assessing the negative impact of delays in applying preventive precautions against the spread of the disease used a six compartmental SIR model with three classes of the infectious: infected without symptoms, infected with symptoms, and infected with complications. They also have hospitalized and recovered classes separately. These authors also engaged an optimal control in the model with four controls that represent the awareness program through sensitization and prevention, quarantine and health monitoring, diagnosis, monitoring and treatment and psychological support with follow-up at time t. The source of SARS-Cov-2 virus is traced to animals, and pangolins, mammals of the order Pholidota, are a major suspect. Therefore a pangolin-human (PH) model was constructed by Ullah *et al.* [45] who have created separate susceptible, exposed, infected and recovered classes both for pangolins and for humans, and also classes for asymptomatic humans and the pangolin bootleg reservoir. This is the only 'vector' type study under consideration which used separate classes for humans and the animal source of the virus. Some authors studied the impact of Covid in reference to other comorbidities. Kouidere *et al.* [46] considered a mathematical model that described the dynamic of spread of COVID-19 pandemic by highlighting the negative impact of applying quarantine total on diabetes people in a given population. They have a complied $SEIRQ_T PDC$ model where the first five classes referred to COVID scenario whereas the classes PDC denoted people in the pre-diabetic state, diabetics without complication and diabetics with complication restively.

Fractional order differential equations

Many authors under the current review have used fractional order differential equations (FDEs) in compartmental models and demonstrate that the same equations capture the real data more accurately than the integer-order model. Ameen *et al.* [1] introduce FDEs with Caputo fractional derivative of order α, $0< \alpha <1$. They use eight compartments, namely, the susceptible individuals (S) in the free environment, latent individuals (L), the traced latent individuals (L_ρ), the suspected individuals (P), the diagnosed individuals (D), the traced susceptible individuals who had direct contact with diagnosed or suspected individuals (S_ρ), the infectious individuals in the free environment (I) and the recovered individuals (R). Raslan *et al.* [32] in their epidemic prediction study for Egypt use an SEQHIR model with fractional derivatives in the Caputo sense. Askar *et al.* [11] in their study of the impact of the lockdown in India use SITR model with Caputo derivatives. For the numerical simulation of the fractional-order system, the authors apply the Adams-Bashforth-Moulton type predictor-corrector scheme. Proportional refuge was applied in the model to study the effect of lockdown in India, which the authors claim is a new concept. Mohammadi *et al.* [40] modified an SIRD model to a fractional order one replacing the ordinary derivatives with the Caputo-type fractional operators and their numerical simulations for data from Japan and Iran showed that the fractional order model followed the real data better. Nissar *et al.* [2] studied a previously proposed integer order SIRD model in the Caputo fractional derivative mode. Panwar *et al.* [4] however used two approaches: Caputo–Fabrizio (CF) and Atangana-Baleanu-Caputo (ABC) approaches and compared the effectiveness of an SEIR model to find notable differences under identical parameter values.

This is ascribed to the memory properties of the kernel in the definitions of the fractional operators because the CF derivative has an exponential kernel, whereas the ABC approach uses a generalized Mittag-Leffler kernel. They also showed that the graphical results demonstrated the CF approach to provide better suitability for mild cases whereas, the ABC approach provides superior and more flexible results for critical cases. A fractal-fractional approach in the Caputo sense with power law was adopted by Abdulwaasa *et al.* [13] for studying the outbreaks in India with 5 compartments SEIRV (V standing for concentration COVID-19 in the surrounding environment) and which was solved with the help of fractional Adams Bashforth (AB) method. Ali *et al.* [6] also used a similar approach with eight compartments including susceptible, exposed, infected, super spreaders, asymptomatic, hospitalized, recovery and fatality classes and applying AB method to simulate the results. Sinan *et al.* [47] used SEIQR fractional mathematical model with harmonic mean type incidence rate and treatment. For numerical results they used Homotopy Perturbation Method (HPM). An SEIR model was constructed by Ali *et al.* [17] with Caputo derivatives numerical simulations of which were done suing Laplace Adomian decomposition method and stability analysis using Ulam-Hyers-Rassians stability method (UHR). Bushnaq *et al.* [31] also modeled COVID using SEIR Caputo sense fractional derivatives and with optimal control strategies, the adjoint system of which involves Riemann–Liouville derivatives. Ndairou *et al.* [30] in their studies also used Caputo type derivatives for the SEIPAHRF model where I, A and P classes referred to the symptomatic, symptomatic and super spreaders respectively and F referred to the fatality/dead cases. They have solved the model using a subroutine called FracPECE to approximate numerically the solution of the model.

Other methods

Apart from the integer order and fractional order differential equations, there are other modeling methods also being used by authors. Kumar *et al.* [9] in their study of COVID in India, proposed the use of generalized polynomial regression of order 6 and discussed the function equations for cumulative infections, cumulative recoveries, cumulative deaths and cumulative active infections respectively as a function of time. Sharma *et al.* [10] adopted a correlation technique based on mediative fuzzy logic based on which technique, they predicted the cases in India by the end of December 2020. The authors perceived mediative fuzzy logic to play a vital role in the analysis of COVID cases where there are some conflicting parameters in the analysis of the model related to their interconnecting nature. A compartmental stochastic time-delay dynamic model (FUDAN-CCDC model) based on China Center for Disease Control and Prevention (CCDC) data was proposed by Xiao *et al.* [3] in their model to predict the confirmed cases in Wuhan and the model was built on a previous time lag dynamics model (TDD-NCP), which clearly considered the time lag effect of the incubation period on the spread of COVID. Claiming stochastic models to be more appropriate than the deterministic models, Danane *et al.* [26] considered a stochastic SIQR model driven by Levy noise. Adak *et al.* [29] however proposed two models: first they propose a deterministic SLIRD model (where L stands for the latent class) and then extend it to a stochastic model by incorporating the stochasticity through parameter perturbation techniques. In comparison of the methods, the authors have found that the stochastic system has no interior equilibrium point but the deterministic system has and the solution of the stochastic system fluctuates around the deterministic solution, however, both solutions become identical as the noise intensity becomes too low. A delay differential equation (DDE) model tailored to incorporate different types of immune response, namely, short-lived immunity of all types, and short-lived sterilizing immunity with durable severity-reducing immunity, was proposed by Shayak *et al.* [48]. Yoon *et al.* [37] used interview methods to investigate whether mathematical schemes for comparing the relative sizes of quantities, rate of change, slope, and graph were either productive or unproductive for citizens' (the study sample was 25 US and 7 South Korean citizens) attempts to interpret COVID-19 data.

They also explored how people used those schemes in a complex and time-sensitive situation and illustrated how citizens' other knowledge and beliefs about news sources, uncertainty, and the reliability of scientific expertise and data collection interacted with their mathematical schemes as they interpreted COVID-19 data. Omar *et al.* [33] modeled COVID dynamics with three mathematical dynamic models, fractional order modified SEIRF (F referring to the death class) model (with Caputo derivative), first order stochastic modified SEIRF model (adding white noise terms satisfying Wiener process properties), and fractional order stochastic modified SEIRF model, to characterize and predict virus behavior. The fractional order modified SEIRF model is solved numerically using Euler's method (with step time equals 0.001 day) and the stochastic models are solved numerically using Euler - Murayama method (with number of paths equals 10 and discretization of time equals 0.001 day, then the average path solution is taken). Martonosi *et al.* [49] used optimization and game theoretic approaches to model the COVID-19 U.S. vaccine market (Pfizer-BioNTech and Moderna). The coinfection of SARS-Cov-2 with influenza was studied with models [52] constructed using SimBiology toolbox from Mathlab.

FINDINGS AND RECOMMENDATIONS FOR COVID CONTROL

In this section, we briefly summarize the findings and recommendations for COVID prevention and control suggested by the studies under consideration. These recommended measures are mostly non-pharmaceutical as most of the studies belonged to the pre- vaccination period. A model for Nepal [18] showed that the lockdown level that can reduce the contact rate by 50% will decrease the peak of new cases of COVID-19 by more than 95% and the cumulative cases by more than 85%. Similarly, the total COVID-19 cases can be reduced more than 70% if the detection and isolation policy is increased by 1.4 times the base case. The model also identified that for a significantly large level of the detection and isolation, the disease spread could be avoided without needing a high level of lockdown. Sharma *et al.* [10] who constructed a mediative fuzzy logic model for 5 states in India predicted the number of infections by the end of 2020 and concluded that if the people followed the instructions used by the government regarding COVID-19 and go along with the isolation process in the society and observe a proper instruction chart and if the people include a hygienic diet plan, then it may reduce the spread rate of COVID-19 pandemic in the globe. Askar *et al.* [11] found that the effect of lockdown on the basic reproduction number is very small and hence the authors recommended preventive measures like physical and social distancing as the only option to halt the transmission of the disease. In a model [15] which used real data from Pakistan also it was concluded that the methods used by the public health sector like social distancing, use of face mask and hand sanitizer are more successful with a controlling transmission rate. The model [40] which considered data from Japan and Iran used social distancing as a control strategy and found that this strategy was highly effective in both the countries. Gumel *et al.* [35] in their primer employed various simulations with data from US and found out that the elimination of the pandemic is greatly enhanced by combining face mask strategy along with other strategies like lock down/social distancing. They found that if lockdown and social distancing measures could reduce community transmission by 55%, the use of even low efficacy cloth masks by at least 57% of the population could pave for the elimination of the disease, whereas if the masks were high efficacy ones (like medical/surgical/N95), the percentage of the users suffice would be as low as 28%. Apart from the above strategies, the importance of awareness and enlightenment programs [20] among the most vulnerable communities especially in countries with poor health care systems was found out to be critical against the background of Nigeria. Chhetri *et al.* [50] in their optimal control within host model on crucial inflammatory mediators and drug interventions concluded that 'the interventions in immune modulation do affect the way of spread of infection but they become effective only along with the drugs preventing viral replication'. Yamomonto *et al.* [36] who quantified compliance with Covid mitigation policies in the US counted social distancing policy and its compliance to be of top priority in reducing the number of cases and arresting the spread of the disease. Din *et al.* [16] using their SIR model with convex incidence rate recommended the prohibition of migration and isolation of infected ones for containing the disease. Foy *et al.* [12] who studied vaccine allocation strategies in India recommended prioritized vaccination for older age groups (>60 years) for reduction in cumulative deaths. Dwomoh *et al.*

[24] emphasized multiple intervention in the context of Ghana, namely, 'Effective and enhanced contact tracing, provision of PPEs, improved case management, social distancing, use of face mask, intensive media coverage, personal hygiene, and immunity-boosting'. The coupled socio-epidemiological Covid model [41] which developed strategies to prioritize Covid vaccines in the context of data from Ontario predicted the number of deaths averted as per these strategies. The authors were of opinion that 'for later vaccination start dates, use of vaccines to interrupt transmission might prevent more deaths than prioritizing vulnerable age groups'. Creation of a special "protected class" [25] which consist of individuals who must provide sufficient evidences that their environment is risk-free from the infection is another recommendation, since researchers who compared various strategies concluded that investing in the protected class is the best strategy since it reduces the most number of infections. In Nigerian context, the models and simulations proposed by Ahmad *et al.* [22] recommend to the 'authorities and health practitioners in Nigeria to work hard to ensure that the contact between the exposed individual and susceptible individuals is minimized'. The model constructed by Bai *et al.* [8] based on Shenzhen data advocated for 'strict control of imported cases and extensive use of facial masks'. In the context of the Indian lock down, it was proposed [14] that even though complete lock down is mandatory for reducing the infection to zero, in the context of the financial crisis it may cause in future, emphasis should be given for other preventive measures like increase in rate of contact tracing and quarantining the confirmed cases of COVID-19 on a regular basis. Modeling studies [38] have also suggested that even though vaccination helps to reduce Covid deaths substantially it only provides partial protection for the individual and hence warned against early or rapid relaxation of Non Pharmaceutical Interventions. In the background of international travel restrictions imposed by various countries, models [42] have found that as per the estimated 2020 travel volumes, imported cases in September 2020, accounted for no more than 10% of total incidence in 125 countries and less than 1% in 44 countries. Hence researchers suggested that "stringent travel restrictions might have little impact on epidemic dynamics except in countries with low COVID-19 incidence and large numbers of arrivals from other countries, or where epidemics are close to tipping points for exponential growth". Serbian models [51] predicted that 'the disease has a potential to linger in population and that it will most probably have a seasonal pattern'. They also suggested that an 87% vaccine coverage was enough for arresting the virus circulation, with expected R_0 as 2.46 and vaccine efficacy of 68%. The vaccine pricing model [49] in the US context yielded realistic prices, averaging $34–$35 per two-dose regimen. The study also suggested that the CDC could negotiate prices with the manufacturers that keep public sector prices as low as possible.

CONCLUSION

History has witnessed many deadly pandemics like Plague, Cholera, Influenza, etc. at various moments. The present generation is worst affected by Covid 19, a corona virus disease, which has been engulfing both human life and living for the past 2 years. Governments, Health Organizations, NGOs, and Scientists all have been responding their level best to eradicate this pandemic. There is no doubt that mathematical modeling studies also contribute in various capacities for the study of the disease spread and its control/eradication. Researchers use various mathematical tools and techniques for finding out strategies to bring the basic reproduction number R_0 to less than 1 when the disease will die out. The present paper aims at a basic survey of about 50 early models for Covid 19 developed by researchers from various countries. The country wise distribution and the data used in these papers, the various mathematical methods employed, findings and recommendations proposed for Covid control are all elaborated. It is observed that most of the existing mathematical modeling tools, ranging from deterministic to stochastic, integer order to fractional order, graph theoretic to game theoretic, statistical to fuzzy, have all been used to model the current pandemic, predict its future course and progression and recommend control measures. It is hoped that the findings and recommendations of modeling studies will guide policy makers in chalking out effective and optimal disease control measures.

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