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Research Article

INVESTIGATING FORCES ON A STATIONARY DIAMAGNETIC OBJECT IN MAGNETIC FIELDS THROUGH A THEORETICAL APPROACH

[1,] *Phan Thái An, [2]Nguyễn Đức Khoa and [3]Vũ Thế Sơn

[1]School of Electrical and Electronic Engineering, Hanoi University of Science and Technology, Vietnam
[2, 3]School of Information and Communication Technology, Hanoi University of Science and Technology, Vietnam

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Abstract

When a diamagnetic object is being placed in a magnetic field, a phenomenon occurs as an electromagnetic force appears despite the object being stationary. This is interesting, because as we know, generally magnetic force occurs when an electric charge moves, which seems to be not the case here. This unexpected behavior caught our interest, leading to the production of this paper. In this study, we will try to interpret why this phenomenon happens, the condition of which it occurs (i.e whether this occurs if the material being paramagnetic instead) and general equations of the force through a theoretical model, therefore broadening our understanding of how a stationary object interacts with magnetic fields.

Keywords: Diamagnetic, Magnetic field.

INTRODUCTION

Traditionally, it is often understanded that magnetic fields can only affect moving electric charge. So why can a stationary diamagnetic object interact with them? It seems to be related to the nature property of diamagnetic material, in which it creates an opposing magnetic field when interacting with external one. To interpret this, often there are two models: an atomic current model can be used; for a higher level, a magnetic dipole moment can be utilized. In both cases, an internal electric current has occurred, therefore partially explaining the problem through logic. The atomic current model is very complicated and often time is not very distinct from the magnetic dipole moment one, especially in macrophysics when the differences are negligible (magnetic dipole moment is the approximate model of a ring of electric current when being small enough). Therefore, we will use the magnetic dipole moment model, which is often time accurate enough for the macro world, for simplification.

METHODOLOGY

First, we consider the appearance of the magnetic field B in the r-axis when the magnetic field varies along z-axis:

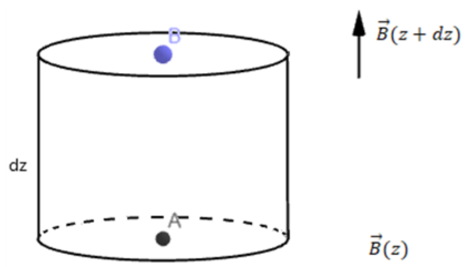

Figure 1.

Corresponding Author:** ***Phan Thái An,
School of Electrical and Electronic Engineering, Hanoi University of Science and Technology, Vietnam.

The magnetic flux passing through the bottom surface of the cylinder:

$Ø_{(z)} = B_{(z)}.\pi.r^2$

The magnetic flux passing through the top surface of the cylinder:

$Ø_{(z+dz)} = (B_{(z)}+dB_{(z)}).\pi.(r+dr)^2$

The magnetic flux passing through the side of the cylinder:

$Ø_{(r)} = B_{(r)}.2\pi r.dr.$
Because flux is conserved, we have: $Ø_{(z)} = Ø_{(z+dz)} + Ø_{(r)}$
$\Rightarrow B_{(z)}.\pi.r^2 = = (B_{(z)}+dB_{(z)}).\pi.(r+dr)^2 + B_{(r)}.2\pi r.dr$

Ignore the infinitesimals $(dr)^2$, we have:
$B(r) = -\frac{r}{2}.\frac{dB(z)}{dz}$

- Then, we consider three situations for homogeneous object before giving the general equations:

1. A cylinder:

We divide the original cylinder into thin and thick cylindrical with layers dz between (r, r+dr)

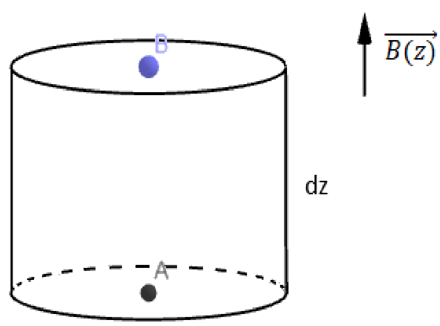

Figure 2.

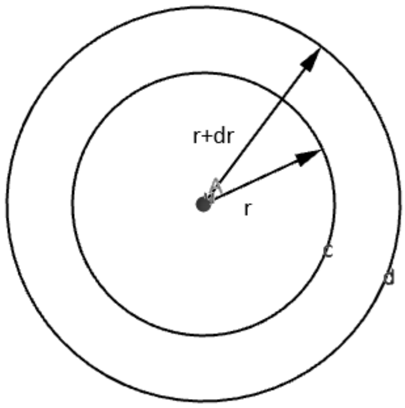

Figure 3.

The volume of this part : $dV = 2\pi r.dr.dz$

The magnetic moment of this part: $dm = \chi.H.dV = \chi.\frac{B(z)}{\mu 0}.2\pi r.dr.dz = d_{I(x,y)}.\pi.r^2$

$\Rightarrow d_{I(x,y)} = 2.\ \chi.\ \frac{B(z)}{\mu 0}.\ \frac{dr}{r}.\ dz$ ($d_{I(x,y)}$ is the circular current at position of radius r)

$B(r) = -\frac{r}{2}.\frac{dB(z)}{dz} \Rightarrow dF_{(z)} = d_{I(x,y)}.\ B_{(r)}.2\pi r = -\ 2\pi r.\ \chi.\ \frac{B(z)}{\mu 0}.\ dr.\ dB_{(z)}$

$\Rightarrow F_{(z)} = -\frac{2\chi}{\mu 0}.\int_0^r \pi.rdr\int_0^z B(z)dB(z) = -\frac{\chi}{\mu 0}.\int dx.dy\int B(z)dB(z)$

2. A sphere:

We divided the original sphere into thin spherical shells located at positions (r, r+dr).

We divided the sphere into ring determined by (θ, θ +dθ).

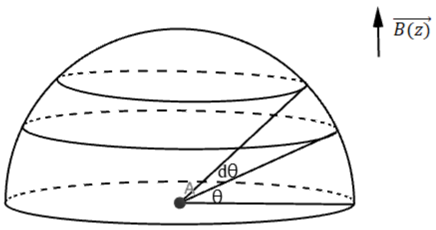

Figure 4.

The volume of this part: $dV = 2\pi r.\cos\theta.r.d\theta.dr = 2\pi.r^2.dr.\cos\theta.d\theta$

The magnetic moment of this part: $dm = \chi.H.dV = \chi.\frac{B(z)}{\mu 0}.2\pi.r^2.dr.\cos\theta.d\theta = d_{I(x,y)}.\pi.(r.\cos\theta)^2$

$\Rightarrow d_{I(x,y)} = 2.\ \chi.\ \frac{B(z)}{\mu 0}.\ \frac{d\theta}{\cos\theta}.\ dr$ ($d_{I(x,y)}$ is the circular current at position of ring)

$B(r) = -\frac{r}{2}.\frac{dB(z)}{dz} \Rightarrow dF_{(z)} = d_{I(x,y)}.\ B_{(r)}.2\pi r = -2\pi r.\ \chi.\ \frac{B(z)}{\mu 0}.\ \frac{dr.\cos\theta}{dz}.\ dr.\ dB_{(z)} = -2\pi r.\ \chi.\ \frac{B(z)}{\mu 0}.\ dr.\ dB_{(z)}$ (because $dz = r.\cos\theta.d\theta$)

$\Rightarrow F_{(z)} = -\frac{2\chi}{\mu 0}.\int_0^r \pi.rdr\int_0^z B(z)dB(z) = -\frac{\chi}{\mu 0}.\int dx.dy\int B(z)dB(z)$

3. Random shape:

Considering a small cube at coordinates (x,y,z)
The volume of this part: $dV = dx.dy.dz$

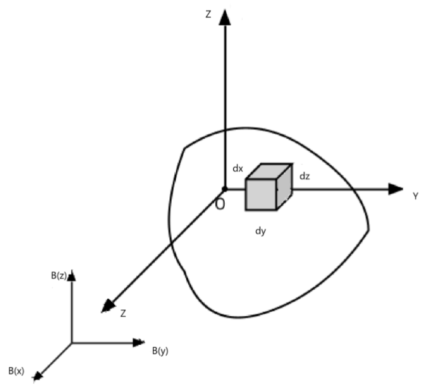

Figure 5.

The magnetic moment of this part: $dm = \chi.H.dV = \chi.\frac{B(z)}{\mu 0}.dx.dy.dz = d_{I(x,y)}.dx.dy$

$\Rightarrow d_{I(x,y)} = 2\chi.\ \frac{B(z)}{\mu 0}.\ dz$ ($d_{I(x,y)}$ is the circular current at position of xy plane)

The magnetic flux passing through this piece: $d\emptyset_{(z)} = dB_{(z)}.dx.dy$

The "magnetic moment" potential energy of this piece:
$dW_t = d_{I(x,y)}.d\emptyset_{(z)} = \chi.\frac{B(z)}{\mu 0}.dB_{(z)}.dx.dy.dz$

The magnetic force acting on this piece: $dF_{(z)} = -\frac{\partial dWt}{\partial z} = \chi.\frac{B(z)}{\mu 0}.dB_{(z)}.dx.dy$

$\Rightarrow F_{(z)} = -\frac{\chi}{\mu 0}.\int dx.dy\int B(z)dB(z)$

Similarly:

$F_{(y)} = -\frac{\chi}{\mu 0}.\int dz.dx\int B(y)dB(y)$
$F_{(x)} = -\frac{\chi}{\mu 0}.\int dz.dy\int B(x)dB(x)$

- Finally, we prove the equation for general cases:

Considering a small cube at coordinates (x,y,z)
The volume of this part: $dV = dx.dy.dz$

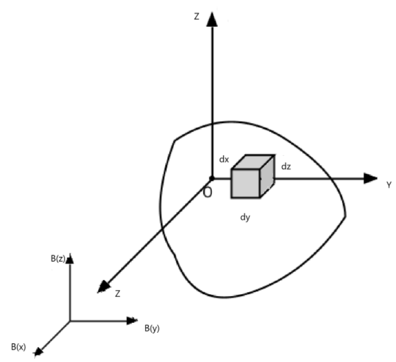

Figure 6.

The magnetic moment of this part: $dm = \chi_{(x,y,z)}.H.dV = \chi_{(x,y,z)}.\frac{B(z)}{\mu 0}.dx.dy.dz = d_{I(x,y)}.dx.dy$

$\Rightarrow d_{I(x,y)} = 2\ \chi_{(x,y,z)}.\ \frac{B(z)}{\mu 0}.\ dz$ ($d_{I(x,y)}$ is the circular current at position of xy plane)

The magnetic flux passing through this piece: $dØ_{(z)} = dB_{(z)}.dx.dy$

The "magnetic moment" potential energy of this piece:

$$dW_t = d_{I(x,y)}.dØ_{(z)} = \chi_{(x,y,z)}.\frac{B(z)}{\mu 0}.dB_{(z)}.dx.dy.dz$$

The magnetic force acting on this piece: $dF_{(z)} = -\frac{\partial dWt}{\partial z} = \chi_{(x,y,z)}.\frac{B(z)}{\mu 0}.dB_{(z)}.dx.dy$

$$\Rightarrow F_{(z)} = -\int dx.dy \int \frac{\chi(x,y,z)}{\mu 0} B(z).dB(z).$$

So, we have the general equations:

$$F_{(z)} = -\int dx.dy \int \frac{\chi(x,y,z)}{\mu 0} B(z).dB(z).$$

$$F_{(x)} = -\int dz.dy \int \frac{\chi(x,y,z)}{\mu 0} B(x).dB(x).$$

$$F_{(y)} = -\int dx.dz \int \frac{\chi(x,y,z)}{\mu 0} B(y).dB(y).$$

RESULT AND DISCUSSION

For the aforementioned equation, we use the magnetic dipole model to solve the problem. This involves diamagnetic material's nature of creating new magnetic dipoles to create an opposition magnetic field to the outer one. Those magnetic dipoles interact with the external magnetic field; therefore, the object experiences electromagnetic force.

We believe that this model can also work with paramagnetic material; although different from the diamagnetic one, this material already contains dipole magnetic at the beginning, and the occurrence of external fields only makes those dipoles reposition themselves. However, because at the initial state, the dipoles distribute randomly, thus the "average" magnetic dipole vector can be said to be zero. Therefore, diamagnetic and paramagnetic material may behave the same in this particular problem.

One of the atomic electric current models we also consider looks like this:

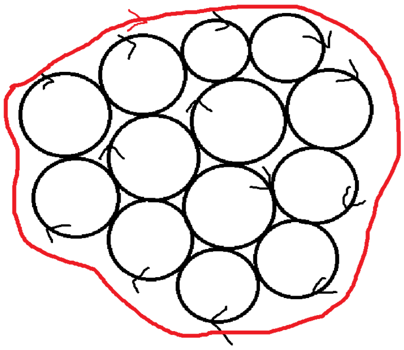

Figure 6.

Each ring represents a magnetic dipole at the microscopic level, which is an electric current loop. Many of these current loops occur close together; all run in the same direction (because all magnetic dipoles can be said to be in the same direction when there is an external field); therefore, the current cancels out inside the material (due to the fact that where those loop currents make contact, the currents run in the opposite directions). As a result, at a macroscopic level, only the surface current (the red one) is retained. We can try to use this current to calculate the force being generated. However, this model fell short because of two problems:

1. The magnetic field vector **B** direction and magnitude can be varied (in other words, grad($\vec{B}$)$\neq$ 0) thus the magnetic dipole vectors are not pointed in the same direction, therefore the internal current remains.
2. When the material itself varies, the individual current values can be different from each other, so the internal current does not cancel out.

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